MATH 279 Test 2 Review

The test will cover sections 3.1 to 4.2 inclusive.

You should be able to:

1. Recognizing when a second order linear differential equation is homogeneous.
2. Solving second order homogeneous linear differential equations with constant coefficients. Here the characteristic equation can have either real distinct roots, or a real repeated root, or complex roots. Remember in the case of a repeated root r, the general solution is $C_1e^{rt} + C_2te^{rt}$.
3. Know the relationship between the Wronskian and linear independence.
4. Find the largest interval on which the solution of a second order linear differential equation exists (applies only to initial value problems).
5. Reduction of order. This is where one finds a second solution to a second order linear homogeneous differential equation, when one solution is already known. If $y = \alpha$ is a solution to the equation $y'' + p(t)y' + q(t)y = 0$, then one looks for a second solution in the form $y = u\alpha$, where u is a function of t to be determined.
6. Method of undetermined coefficients (this will definitely be on the test somewhere). This is for solving a nonhomogeneous linear differential equation with constant coefficients.
7. Variation of parameters. This is for solving a nonhomogeneous linear differential equation where the coefficients need not be constant. If y_1 and y_2 are solutions to the complementary equation, then look for a particular solution in the form $y = uy_1 + vy_2$, where u and v are functions of t to be determined, and impose the condition $u'y_1 + v'y_2 = 0$.

   For example if the equation was $y'' - 2y' + 2y = \ln t$, then the roots of the characteristic equation are $r = 1 \pm i$, so $y_h = e^t(A\cos t + B\sin t)$ and we look for a solution of the form $ue^t\cos t + ve^t\sin t$ where u, v are functions to be determined. Also we impose the condition $u'e^t\cos t + v'e^t\sin t = 0$. If the method of undetermined coefficients works, then so will variation of parameters, but will probably be longer. Also a problem in the test could be phrased so that you were forced to use one method or another.
8. Solving spring-mass-damper systems. Use the equation $mu'' + \gamma u' + ku = F(t)$ where m is the mass, γ is the damping coefficient, k is the spring constant, and F(t) is some external forcing function. Make sure that your units are all the same. Be able to find the amplitude and the period of the vibration and be able to distinguish between underdamped, critically damped and overdamped systems. Be able to find the steady state solution of a forced system.
9. Know how to solve higher order differential equations. Still use characteristic equation but now you will have more solutions (ie. As many as the order of the equation).
10. Know how to find all the solutions to an expression such as $x^6 - 1 = 0$.