Math 267 Test 2 review problems

Directions The test will cover chapter 3. Study these problems, the examples in your notes, and the homework.

Multiple Choice

1. Let $y(t)$ be the solution of

$$y'' + y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

   Then as $t \to \infty$,

   (a) $y(t) \to \infty$
   (b) $y(t) \to 0$
   (c) $y(t) \to 1$
   (d) The limit of $y(t)$ can not be determined from the information given.

2. Noting that $y_1(t) = e^t$ is a solution of

$$ty'' + 2(1-t)y' + (t-2)y = 0,$$

   if you try to find a second solution y_2 in the form $y_2 = e^t v(t)$, which of the following equations must be satisfied by v?

   (a) $v'' + tv' + v = 0$
   (b) $tv'' + 2(1-t)v' + (t-2)v = 0$
   (c) $tv'' + 2v' = 0$
   (d) $tv' + v = 0$

3. The general solution of $y^{(5)} + y''' = 0$ is

   (a) $c_1 + c_2e^t + c_3e^{-t} + c_4te^{-t} + c_5t^2e^{-t}$
   (b) $c_1 + c_2t + c_3\cos t + c_4\sin t$
   (c) $c_1 + c_2t + c_3t^2 + c_4\cos t + c_5\sin t$
   (d) $c_1 + c_2t + c_3t^2 + c_4e^{-t} + c_5te^{-t}$

4. Four solutions of $y^{(4)} - y = 0$ are

   (a) $e^t,\ te^t,\ t^2e^t,\ t^3e^t$
   (b) $e^t\cos t,\ e^t\sin t,\ te^t\cos t,\ te^t\sin t,$
   (c) $e^t,\ te^t,\ e^{-t},\ te^{-t}$
   (d) $e^t,\ e^{-t},\ \sin t,\ \cos t$

5. Let $a < x < b$ denote the longest interval on which the solution of the inital value problem below exists.

$$2(x-1)(x-2)x^2y'' - (x+5)(x-2)xy' - 2(x-1)y = 0; \qquad y(-1) = 3, \;\; y'(-1) = 3$$

Then the interval $a < x < b$ is

(a) $-2 < x < 0$
(b) $-5 < x < 1$
(c) $-5 < x < 2$
(d) $-\infty < x < 0$

6. Suppose that the functions e^{2t} and e^{-3t} form a fundamental set of solutions for the differential equation

$$y'' + a_1y' + a_0y = 0 \qquad (a_0, a_1 \text{ are constants}).$$

Then the coefficient a_0 is

(a) 1
(b) 3
(c) -2
(d) -6

7. Suppose the polynomial equation

$$a_{12}r^{12} + a_{11}r^{11} + \cdots + a^1r + a_0 = 0$$

has solutions r =0, 0, 2, 2, 2, -3, $-3 \pm 2i$, $-3 \pm 2i$, $-3 \pm 2i$. Then the general solution to the differential equation

$$a_{12}y^{(12)} + a_{11}y^{(11)} + \cdots + a^1y' + a_0y = 0$$

is

(a) $y = C_1 + C_2t + C_3e^{2t} + C_4te^{2t} + C_5t^2e^{2t} + C_6e^{-3t} + C_7\cos(2t) + C_8\sin(2t) + C_9e^{-3t}\cos(2t) + C_{10}e^{-3t}\sin(2t) + C_{11}e^{-3t}\cos(2t) + C_{12}e^{-3t}\sin(2t)$

(b) $y = C_1 + C_2t + C_3e^{2t} + C_4te^{2t} + C_5t^2e^{2t} + C_6e^{-3t} + C_7\cos(2t) + C_8\sin(2t) + C_9t\cos(2t) + C_{10}t\sin(2t) + C_{11}te^{-3t} + C_{12}te^{-3t}$

(c) $y = C_1 + C_2t + C_3e^{2t} + C_4te^{2t} + C_5t^2e^{2t} + C_6e^{-3t} + C_7\cos(2t) + C_8\sin(2t) + C_9e^{2t}\cos(3t) + C_{10}e^{2t}\sin(3t) + C_{11}te^{2t}\cos(3t) + C_{12}te^{2t}\sin(3t)$

(d) $y = C_1 + C_2t + C_3e^{2t} + C_4te^{2t} + C_5t^2e^{2t} + C_6e^{-3t} + C_7\cos(2t) + C_8\sin(2t) + C_9e^{-3t}\cos(2t) + C_{10}e^{-3t}\sin(2t) + C_{11}te^{-3t}\cos(2t) + C_{12}te^{-3t}\sin(2t)$

8. Suppose an 8 lb weight is attached to a spring and stretches the spring 1 foot. The system has a damping coefficient of 2.5; the weight is pulled down 8 inches and released.

(a) Write the complete initial value problem and then solve it only far enough to describe the motion of the system with the terminology of this course.
(b) Find the coefficient of resistance for which the motion would be critically damped.

9. A spring-mass-damper system consists of a 10-kg mass attached to a spring with spring constant k=130 N/m; the damper has a damping constant 60 kg/s. At time $t = 0$, the system is set into motion from its equilibrium rest position by giving it an initial downward velocity of 1 m/s.

   (a) Write the complete initial value problem to be solved for $y(t)$, the displacement from equilibrium (in meters). Give numerical values to all constants involved.

   (b) Solve the initial value problem. What is $\lim_{t\to\infty} y(t)$?

10. Suppose a 32 lb weight is attached to a spring and stretches the spring 4 feet. The object is pulled down 6 inches and released. Find the coefficient of resistance for which the resulting motion would be critically damped, then determine the equation of motion by solving the initial value problem.

11. Find the largest interval on which the following initial value problem is guaranteed to have a unique solution:
$$(t^2 - 9)y'' + 2t(t-4)y' + 3ty = t^{-2}e^{-t}, \quad y(2) = -1, \ y'(2) = 3$$

12. Find the largest interval on which the following initial value problem is guaranteed to have a unique solution:
$$4(t^2 + 2t - 15)y'' + 2y' + dfract - 7t - 2y = 0, \quad y(-3) = -1, \ y'(-3) = 3$$

13. Let $a < t < b$ denote the longest interval on which the solution of the initial value problem below exists.
$$2(t-1)^2y'' - 2ty' - 3t(t-4)y = t^{-2}e^{-t}; \qquad y(-1) = 3, \ \ y'(-1) = 3$$
Then the interval $a < t < b$ is

14. Use variation of parameters to find the general solution to the differential equation $y'' + \frac{y'}{t} - \frac{y}{t^2} = 1$ given that $y_1(t) = t$ and $y_2(t) = t^{-1}$ are both solutions to the homogeneous equation $y'' + \frac{y'}{t} - \frac{y}{t^2} = 0$.

15. Use variation of parameters to find the general solution to the differential equation $y'' - \frac{2y'}{t} = t$ given that the solution to the homogeneous equation $y'' - \frac{2y'}{t} = 0$ is $y_h(t) = \frac{c_1t^3}{3} + c_2$.

16. Use variation of parameters to find the general solution to the differential equation $y'' + 4y = \sec 2t$.

17. Find the Wronskian of the two solutions $y_1(t) = e^{2t}$ and $y_2(t) = te^{2t}$ of the differential equation $y'' - 4y' + 4y = 0$.

18. If $y(t) = te^t$ is a solution of $y'' - ty' + ty = \alpha e^t$, what is the value of α?

19. The 7th order differential equation

$$y^{(4)} - y''' - 8y'' - 8y' + 4y = 0$$

has characteristic equation

$$(r - (1 + i))^2 (r - (1 - i))^2 = 0$$

in factored form. What is the general solution?

20. The 4th order differential equation

$$y^{(7)} - 5y^{(6)} + 21y^{(5)} - y^{(4)} - 25y''' + 237y'' + 403y' + 169y = 0$$

has characteristic equation

$$(r + 1)^3 (r - (2 + 3i))^2 (r - (2 - 3i))^2 = 0$$

in factored form. What is the general solution?

21. Find α so that the solution of the initial value problem

$$y'' - y' - 2y = 0; \qquad y(0) = 1, \quad y'(0) = \alpha$$

approaches zero as $t \to \infty$

22. (9 pts) One solution of $ty'' - ty' + y = 0$ is $y_1(t) = t$. If a second solution to the equation is $y_2(t) = v(t)t$ then what is the function $v(t)$?

23. (9 pts) One solution of $ty'' - 3ty' + 3y = 0$ is $y_1(t) = t$. Use reduction of order to find the general solution.

24. (9 pts) One solution of $t^2y'' - 4ty' + 6y = 0$ is $y_1(t) = t^2$. Use reduction of order to find the general solution.

25. Find an appropriate form for y_p if the method of undetermined coefficients were used to solve each of the following **(DO NOT SOLVE)**

    (a) $y'' + 3y' + 2y = t^2e^{-t} + e^{-2t}$ **(DO NOT SOLVE.)**

    (b) $y'' + 4y = 3t \sin 2t + 4e^{2t}$ **(DO NOT SOLVE.)**

    (c) $y'' + 4y = t^2e^{-2t} + e^{2t} + t^2 \cos(2t)$ **(DO NOT SOLVE.)**

    (d) $y'' + 6y' + 9y = t \sin 3t + te^{-3t} + t^3$ **(DO NOT SOLVE.)**

    (e) $y'' + y = 3te^{-t} \cos t + t \sin t$ **(DO NOT SOLVE.)**

    (f) $y'' - 2y' + y = (2 + t)e^t + t^3 - 4t^2$ **(DO NOT SOLVE.)**

    (g) $y'' + 2y' + 2y = t^2e^{-t} + 3e^{-t} \cos t$ **(DO NOT SOLVE.)**

    (h) $y'' + 2y' + y = t^2e^{-t} + 3e^{-t} \cos t$ **(DO NOT SOLVE.)**