3.10 Forced Vibrations

The nonhomogeneous case of section 3.6.

In these problems we have some external forcing function driving the system. The forcing function can be almost anything but usually it is periodic.

$$F(t) = F_1 \cos \omega t + F_2 \sin \omega t$$

So our equation looks like:

$$my''(t) + \gamma \; y'(t) + ky(t) = F_1 \cos \omega t + F_2 \sin \omega t$$

and the solution is once again

$$y(t) = y_h(t) + y_p(t)$$

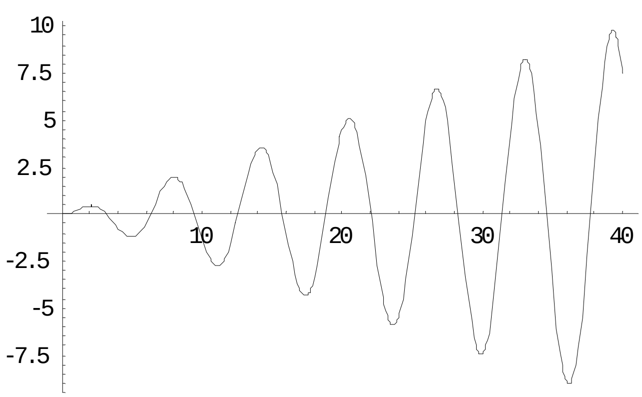
where y_h is the homogeneous solution and y_p is the particular solution.

I. If there is no damping (ie. $\gamma = 0$) then the solutions to (1) are of the form

$$y_h = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

$$y_p = \begin{cases} A\cos \omega t + B\sin \omega t & \text{when } \omega \neq \omega_0 \\ At\cos \omega t + Bt\sin \omega t & \text{when } \omega = \omega_0 \end{cases}$$

The second case ($\omega = \omega_0$) is known as resonance. When the frequency of the forcing function is the same as the natural frequency of the system then the motion of they system is unbounded as $t \to \infty$.



$y = 0.25t\sin t$

II. If γ is not zero then our solutions all have an e^{-rt} component so all the homogenous solutions will go to zero as $t \to \infty$. This part of the solution is known as the transient solution. The total solution is

$$u = u_h + u_p$$

and as $t \to \infty$ the homogeneous part (transient solution) goes to zero so in the long term you are only left with the particular solution. This is called the steady state solution or forced response.

Ex: A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin\left(\frac{t}{2}\right)$ N and moves in a medium that imparts a viscous force of 2N when the speed is 4 cm/sec. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/sec find an expression for the position of the mass at any time *t*. Identify the transient and steady state parts of the solution.

IMPORTANT: make sure your units match up.

g = 9.8 m/s

$$F(t) = 10\sin\left(\frac{t}{2}\right) \text{ kg m/s}^2$$

$$y(0) = 0$$

$$y'(0) = 0.03 \text{ m/s}$$

$$k = \frac{mg}{L} = \frac{(0.5)(9.8)}{0.1} = 490$$

$$\gamma = \frac{2}{0.04} = 50$$

$$my''(t) + \gamma\, y'(t) + ky(t) = F$$

$$5y''(t) + 50y'(t) + 490y(t) = 10\sin\left(\frac{t}{2}\right)$$