Section 5.7 The Delta Function and Impulse Response

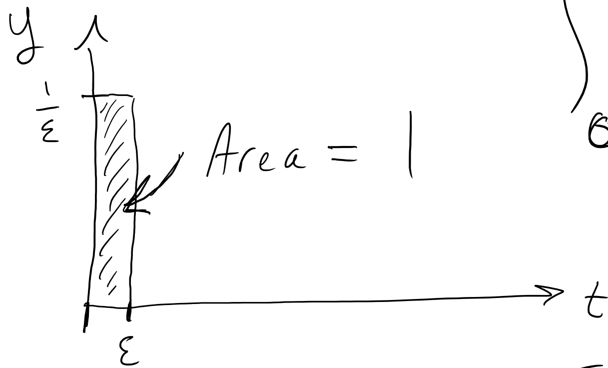
Friday, April 26, 2019 10:07 AM

Define the delta function such that

$$\delta(t)$$

$$\int_a^a \delta(t) = 1$$

$P_\varepsilon(t)$



$$\int_0^\varepsilon P_\varepsilon(t) = 1$$

Impulse force of size F with duration not much $= F\,\delta(t)$

Also

$$\int_a^b f(t)\,\delta(t-t_0)\,dt = \begin{cases} f(t_0) & a \le t_0 \le b \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-t_0 s} \qquad \text{row \#27}$$

Ex:

$$\int_{-2}^{5} (1+e^{-t})\,\delta(t-2)\,dt = \boxed{1+e^{-2}}$$

$t_0 = 2$

Ex:

$$\int_{3}^{5} (1+e^{-t})\,\delta(t-2)\,dt = \boxed{0}$$

Ex:

Ex: $\mathcal{L}\{y'' + 2y' + 2y = \delta(t-1)\}$ $\quad y(0) = y'(0) = 0$

$\mathcal{L}\{\delta(t-a)\} = e^{-as}$

16. $f'(t)$, with $f(t)$ continuous with $\lvert f'(t)\rvert \le Me^{at}$	$sF(s) - f(0)$
17. $f''(t)$, with $f'(t)$ continuous with $\lvert f''(t)\rvert \le Me^{at}$	$s^2F(s) - sf(0) - f'(0)$

$$s^2 Y(s) + 2sY(s) + 2Y(s) = e^{-s}$$

$$Y(s)(s^2 + 2s + 2) = e^{-s}$$

$$Y(s) = \frac{e^{-s}}{s^2+2s+2} = \frac{e^{-s}}{s^2+2s+1+1}$$

$$Y(s) = e^{-s}\frac{1}{(s+1)^2+1}$$

row 14 AND row 12

$\omega = 1$
$\alpha = -1$

12. $e^{\alpha t}\sin\omega t$	$\dfrac{\omega}{(s-\alpha)^2+\omega^2}$
13. $e^{\alpha t}\cos\omega t$	$\dfrac{s-\alpha}{(s-\alpha)^2+\omega^2}$
14. $f(t-\alpha)h(t-\alpha)$, $\quad(\alpha \ge 0)$, with $\lvert f(t)\rvert \le Me^{at}$	$e^{-\alpha s}F(s)$

$\alpha = 1$

$F(s) = \dfrac{1}{(s+1)^2+1}$

row 12: $f(t) = e^{-t}\sin t$

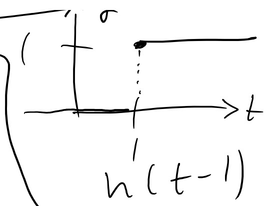
$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{e^{-s}\frac{1}{(s+1)^2+1}\right\}$$

$$\boxed{y(t) = e^{-(t-1)}\sin(t-1)\,h(t-1)}$$

$0 < t < 1$

$$y = 0 \qquad 0 \le t < 1$$

$$y = e^{-(t-1)} \sin(t-1) \qquad 1 \le t < \infty$$



$$h\left(\tfrac{1}{2} - 1\right) = h\left(-\tfrac{1}{2}\right)$$