Section 5.4 Periodic Functions and System Transfer Functions

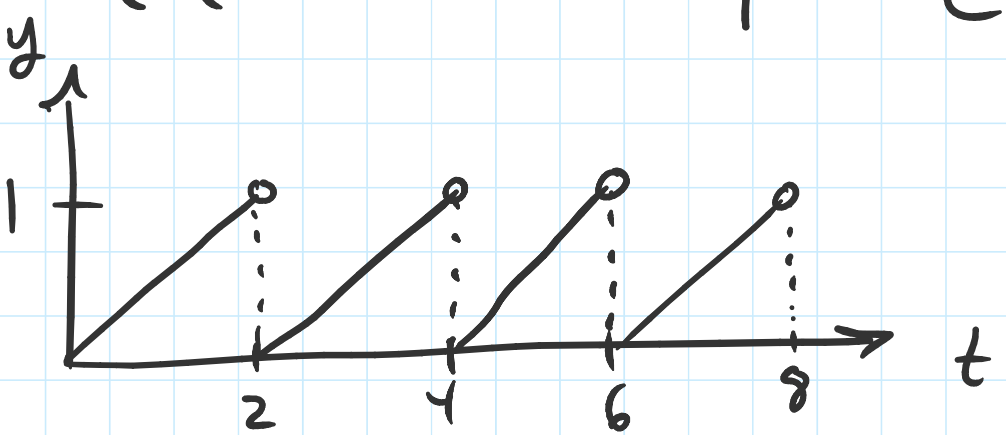
Monday, April 22, 2019 1:31 PM

Let $f(t)$ be a piecewise continuous and periodic function with period T.

Then

$$\mathcal{L}\{f(t)\} = \frac{\int_0^{T} e^{-st} f(t)\,dt}{1 - e^{-sT}}$$

T a number



$$f(t) = \frac{1}{2}t \qquad 0 \le t < 2$$

$$f(t+2) = f(t)$$

$T = 2$ period

$$\mathcal{L}\{f(t)\} = \frac{\int_0^{2} e^{-st}\left(\frac{1}{2}t\right)dt}{1 - e^{-2s}}$$

$1 - e$

$$\frac{1}{2}\int_0^2 t e^{-st}\,dt = \frac{1}{2}\left[-\frac{t}{s}e^{-st} + \frac{1}{s}\int_0^2 e^{-st}\,dt\right]$$

$u = t \qquad dv = e^{-st}\,dt$

$du = dt \qquad v = -\frac{1}{s}e^{-st}$

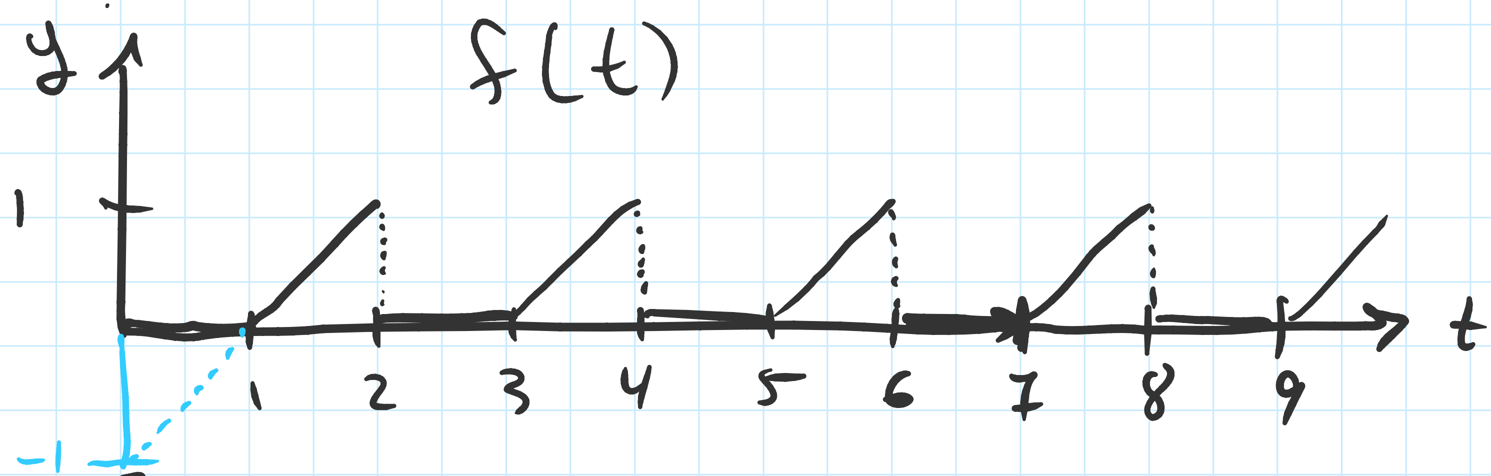
$$= \frac{1}{2}\left[-\frac{t}{s}e^{-st} - \frac{1}{s^2}e^{-st}\right]_{t=0}^{t=2}$$

$$= \frac{1}{2}\left[\overbrace{-\frac{2}{s}e^{-2s} - \frac{1}{s^2}e^{-2s}}^{t=2} - \overbrace{\left(-\frac{1}{s^2}\right)}^{t=0}\right]$$

$$= -\frac{1}{2s^2}\left[2se^{-2s} + e^{-2s} - 1\right]$$

$$\boxed{\mathcal{L}\{f(t)\} = -\frac{(2s+1)e^{-2s} - 1}{2s^2\left(1 - e^{-2s}\right)}}$$

Ex:



$$f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t-1 & 1 \le t < 2 \end{cases}$$

$$f(t+2) = f(t)$$

$$T = 2$$

$$\mathcal{L}\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1 - e^{-sT}}$$

$$\int_0^2 e^{-st} f(t)\,dt = \int_1^2 (t-1)\, e^{-st}\,dt$$

parts $\quad u = (t-1) \quad dv = e^{-st}$