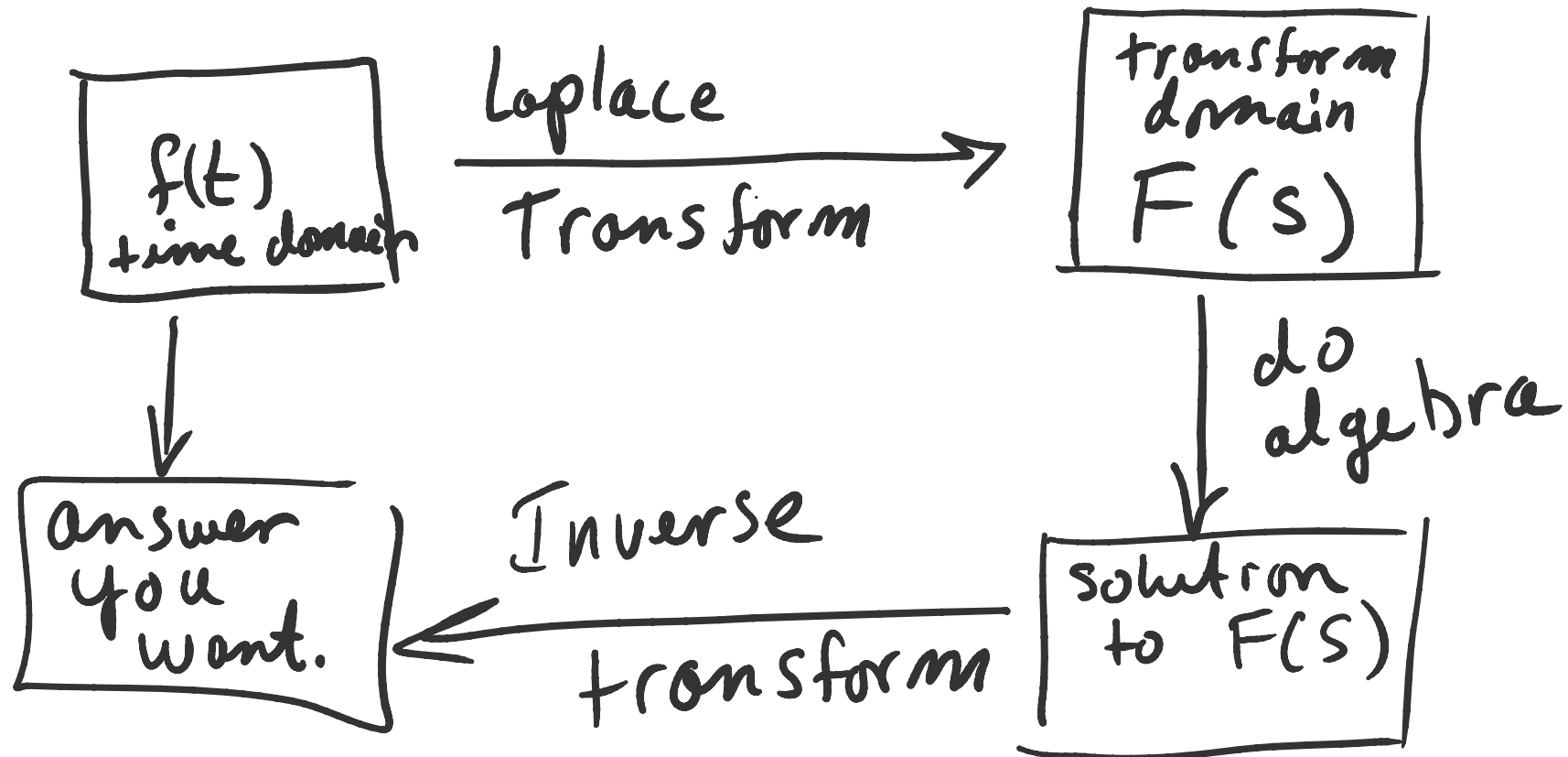


Section 5.1a Laplace Transforms Introduction

Wednesday, April 3, 2019 2:26 PM

Two domains

1. time domain (t)
2. transform domain (s)



Laplace Transform $\mathcal{L}\{f(t)\}$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt = F(s)$$

recall from calc that you need
a limit to solve this.

$$\mathcal{L}\{f(t)\} = \lim_{A \rightarrow \infty} \int_0^A f(t) e^{-st} dt$$

Ex 1: find the Laplace Transform

of $f(t) = 8$

$$\mathcal{L}\{8\} = -\frac{1}{s} \int_0^{\infty} 8 e^{-st} (-s dt) \quad \begin{array}{l} u = -st \\ du = -s dt \end{array}$$

$$= \lim_{A \rightarrow \infty} \left(-\frac{1}{s}\right) \left[8e^{-st} \right]_0^A$$

$$= -\frac{1}{s} \lim_{A \rightarrow \infty} [8e^{-sA} - 8]$$

$$\text{If } \begin{array}{l} s < 0 \\ e^{-sA} \rightarrow \infty \\ s > 0 \end{array}$$

$$\boxed{= \frac{8}{s} \quad \text{for } s > 0}$$

Ex 2: $\mathcal{L}\{t^2\} = \int_0^{\infty} t^2 e^{-st} dt$ $a = -s$
 $n = 2$

Table: $\int t^n e^{at} = \frac{t^n e^{at}}{a} - \frac{n}{a} \int t^{n-1} e^{at} dt$

$$\lim_{A \rightarrow \infty} \int_0^A t^2 e^{-st} dt = \frac{t^2 e^{-st}}{-s} + \frac{2}{+s} \int t e^{-st} dt$$

$$= \frac{t^2 e^{-st}}{-s} + \left(\frac{2}{s}\right) \left[\frac{t e^{-st}}{-s} + \frac{1}{+s} \int e^{-st} dt \right]$$

$$= \lim_{A \rightarrow \infty} \left[\frac{t^2 e^{-st}}{-s} - \frac{2t e^{-st}}{s^2} - \frac{2}{s^3} e^{-st} \right]_0^A$$

$$= \lim_{A \rightarrow \infty} \left[\left(-\frac{t^2}{s} - \frac{2t}{s^2} - \frac{2}{s^3} \right) e^{-sA} + \frac{2}{s^3} \right] \quad s > 0$$

$$= \frac{2}{s^3}, \quad s > 0$$

$$= \sqrt[3]{s}, \quad s > 0$$

$$\underline{\text{Ex 3:}} \quad \mathcal{L}\{e^{3t}\} = \int_0^{\infty} e^{3t} e^{-st} dt$$

$$= \lim_{A \rightarrow \infty} \int_0^A e^{(3-s)t} dt \quad \begin{array}{l} u = (3-s)t \\ du = (3-s)dt \end{array}$$

$$= \lim_{A \rightarrow \infty} \frac{1}{3-s} \int_0^A e^{(3-s)t} (3-s) dt$$

$$= \lim_{A \rightarrow \infty} \frac{1}{3-s} \left[e^{(3-s)t} \right]_0^A \quad \begin{array}{l} 3-s < 0 \\ 3 < s \end{array}$$

$$= \lim_{A \rightarrow \infty} \frac{1}{3-s} \left[e^{(3-s)A} - 1 \right]$$

$$= -\frac{1}{3-s} = \boxed{\frac{1}{s-3}, s > 3}$$

$$= -\frac{1}{3-s} = \boxed{\frac{1}{s-3}, s > 3}$$

Section 5.1b Laplace Transforms Existence and Uniqueness

Friday, April 12, 2019 11:53 AM

Ex: $\mathcal{L}\{e^{t^2}\} = \int_0^{\infty} e^{t^2} e^{-st} dt$

$$= \int_0^{\infty} e^{t^2 - st} dt$$

Diverges.

$$= \int_0^{\infty} e^{t(t-s)} dt$$

Consider $\underbrace{t(t-s)}$ For any $s > 0$
is positive for $t > s$

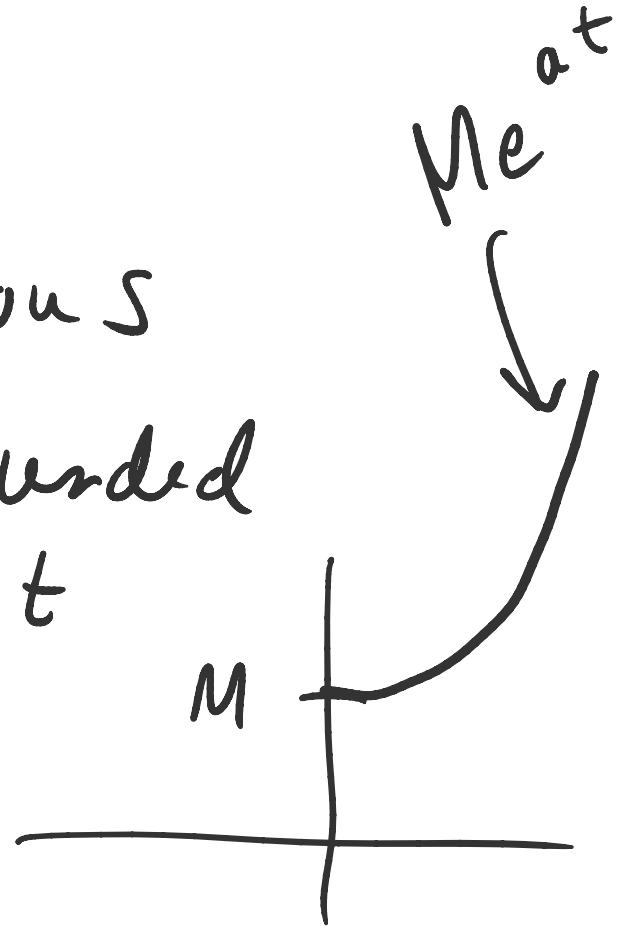
Existence & uniqueness

Need two things

1. Piecewise continuous
2. Exponentially bounded

$$f(t) < Me^{at}$$

$$a > 0$$

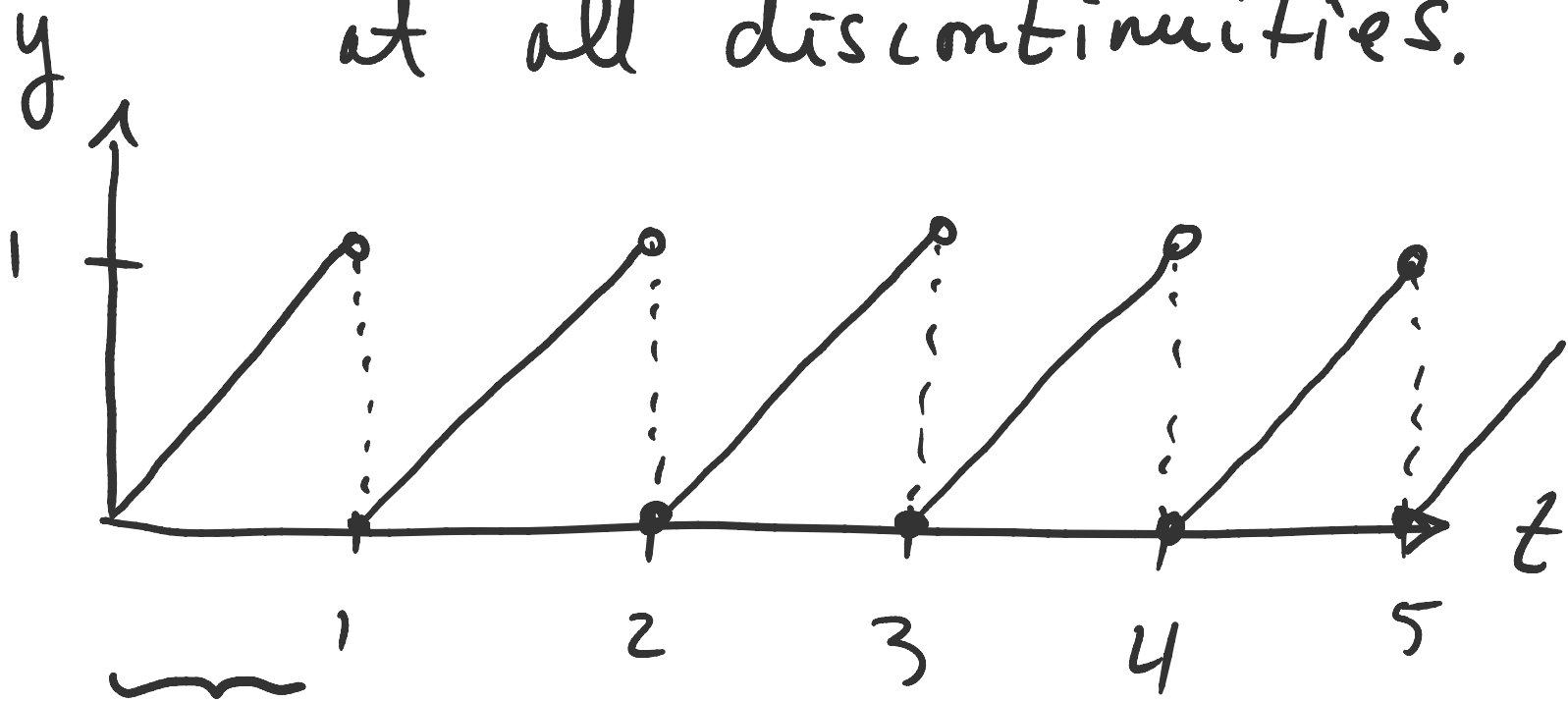


Piecewise continuous

1. Finite number of discontinuities on any finite interval.

on any finite interval.

2. limit from the left and limit from the right exist at all discontinuities.



$$f(t) = t, \quad 0 \leq t < 1 \quad f(t+1) = f(t)$$

Periodic function w/ period = 1

Laplace Rules:

$$f(t) = c_1 f_1(t) + c_2 f_2(t)$$

$$1. \mathcal{L}\{f(t)\} = c_1 \mathcal{L}\{f_1(t)\} + c_2 \mathcal{L}\{f_2(t)\}$$

$$2. \text{ If } f(t) = f_1(t) f_2(t)$$

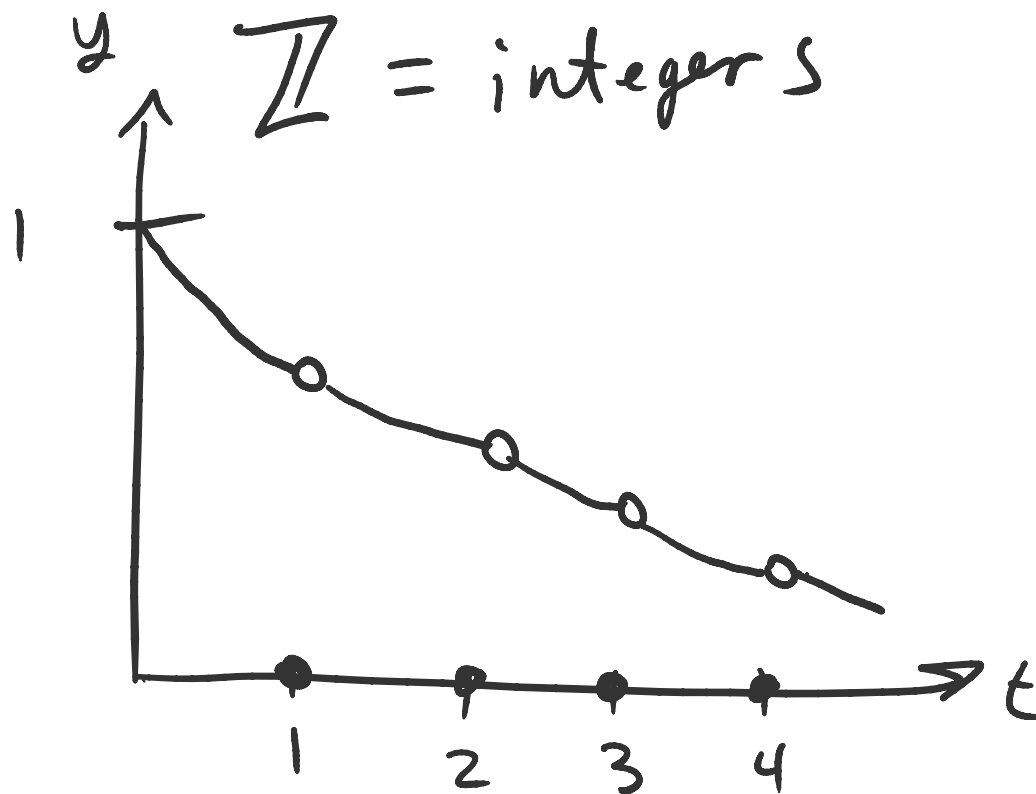
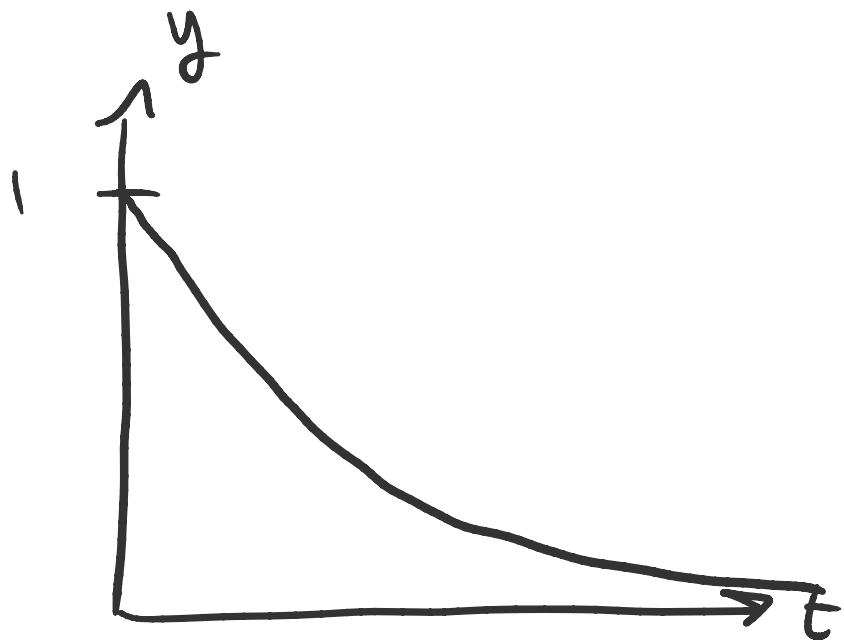
then $\mathcal{L}\{f(t)\}$ exists.

Inverse Laplace & uniqueness

$$f_1(t) = e^{-t}$$

$$f_2(t) = \begin{cases} e^{-t} & t \notin \mathbb{Z} \\ 0 & t \in \mathbb{Z} \end{cases}$$

$\mathbb{Z} = \text{integers}$



$$\mathcal{L}\{f_1(t)\} = \mathcal{L}\{f_2(t)\} = \frac{1}{s+1}$$

$$\mathcal{L}^{-1}\{f_1(t) + f_2(t)\} = \mathcal{L}^{-1}\{f_1(t) + f_2(t)\} = \overline{s+1}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$$

row 4. $\mathcal{L}\{e^{\alpha t}\} = \frac{1}{s-\alpha}$

$$e^{\alpha t} = \mathcal{L}^{-1}\left\{\frac{1}{s-\alpha}\right\}$$

Ex: Find Inverse Laplace of

$$F(s) = \frac{3}{s+2} + \frac{5}{s-2}$$

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left\{\frac{3}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{5}{s-2}\right\}$$

$$= 3 \mathcal{L}^{-1} \left\{ \frac{1}{s+2} \right\} + 5 \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\}$$

$$f(t) = 3e^{-2t} + 5e^{2t}$$