

Table 1: A Brief Table Of Laplace Transform Pairs

$f(t), t \geq 0$	Laplace Transform $F(s)$
1. $h(t) = \begin{cases} 1 & t \geq 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{s} \quad (s > 0)$
2. 1	$\frac{1}{s^2} \quad (s > 0)$
3. $t^n, \quad n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}} \quad (s > 0)$
4. $e^{\alpha t}$	$\frac{1}{s - \alpha} \quad (s > \alpha)$
5. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2} \quad (s > 0)$
6. $\cos \omega t$	$\frac{s}{s^2 + \omega^2} \quad (s > 0)$
7. $\sinh bt = \frac{e^{bt} - e^{-bt}}{2}$	$\frac{b}{s^2 - b^2} \quad (s >  b )$
8. $\cosh bt = \frac{e^{bt} + e^{-bt}}{2}$	$\frac{s}{s^2 - b^2} \quad (s >  b )$
9. $e^{\alpha t} f(t)$ , with $ f(t)  \leq M e^{at}$	$F(s - \alpha) \quad s > \alpha + a$
(10) - (13) are four special cases of (9)	
10. $e^{\alpha t} h(t)$	$\frac{1}{s - \alpha} \quad (s > \alpha)$
11. $e^{\alpha t} t^n, \quad n = 1, 2, 3, \dots$	$\frac{n!}{(s - \alpha)^{n+1}} \quad (s > \alpha)$
12. $e^{\alpha t} \sin \omega t$	$\frac{\omega}{(s - \alpha)^2 + \omega^2} \quad (s > \alpha)$
13. $e^{\alpha t} \cos \omega t$	$\frac{s - \alpha}{(s - \alpha)^2 + \omega^2} \quad (s > \alpha)$
14. $f(t - \alpha)h(t - \alpha), \quad (\alpha \geq 0),$ with $ f(t)  \leq M e^{at}$	$e^{-\alpha s} F(s) \quad (s > a)$
15. $h(t - \alpha), \quad \alpha \geq 0$	$\frac{e^{-\alpha s}}{s} \quad (s > 0)$
16. $f'(t)$ , with $f(t)$ continuous with $ f'(t)  \leq M e^{at}$	$sF(s) - f(0) \quad s > \max\{a, 0\}$
17. $f''(t)$ , with $f'(t)$ continuous with $ f''(t)  \leq M e^{at}$	$s^2 F(s) - s f(0) - f'(0) \quad s > \max\{a, 0\}$

18. $f^{(n)}(t)$ , with $f^{(n-1)}(t)$ continuous with $ f^{(n)}(t)  \leq Me^{at}$	$s^n F(s) - s^{n-1}f(0) - \dots$ $-s f^{(n-2)} - f^{(n-1)}(0)$	$s > \max\{a, 0\}$
19. $\int_0^t f(u) du$ , with $ f^{(n)}(t)  \leq Me^{at}$	$\frac{F(s)}{s}$	$s > \max\{a, 0\}$
20. $\int_0^t f(t - \lambda)g(\lambda) d\lambda$	$F(s) \cdot G(s)$	
21. $t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	$(s > 0)$
22. $t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	$(s > 0)$
23. $\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$	$(s > 0)$
24. $\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)^2}$	$(s > 0)$
25. $tf(t)$	$-F'(s)$	
26. $t^k f(t)$	$(-1)^k F^{(k)}(s)$	
27. $\delta(t - a)$	$e^{-as}$	$(s > 0)$

### For Periodic Functions:

Let  $f(t)$  be a piecewise continuous periodic function defined on  $0 \leq t < \infty$ , where  $f(t)$  has period  $T$ . Then

$$\mathcal{L}\{f(t)\} = \frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}, \quad s > 0$$

### Convolution

$$(f * g)(t) = \int_0^t f(t - \lambda)g(\lambda) d\lambda$$

$$(f * g)(t) = \mathcal{L}^{-1}\{F(s)G(s)\}$$