Section 4.3

Monday, October 3, 2022 1:44 PM

4.3 Basis and linearly independent sets

Recall:

An indexed set of vectors $\{v_1, v_2, \ldots, v_n\}$ is said to be **linearly independent** if the equation

$$a_1v_1 + a_2v_2 + \cdots + a_nv_n = 0$$

has only the trivial solution $a_1 = a_2 = \cdots = a_n = 0$. It is said to be **linearly dependent** otherwise.

Basis

Definition 4.6. Let H be a vector space (including possibly that H is a subspace of some other vector space). A set of vectors

$$\mathcal{B} = \{b_1, b_2, \ldots, b_n\}$$

is a **basis** for H if:

1. $\mathcal{B}$ is a linearly independent set of vectors. (This means there is a pivot position in every column of the reduced matrix consisting of the columns of $\mathcal{B}$).
2. $H = \text{Span}\,\mathcal{B}$.

Not L.I.

$$\begin{bmatrix}1\\1\end{bmatrix} - \frac{1}{2}\begin{bmatrix}2\\2\end{bmatrix} = 0$$

$$a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

a & b = 0

L.I.

Example 4.3.1. Let $v_1 = \begin{bmatrix}3\\0\\-6\end{bmatrix}$, $v_2 = \begin{bmatrix}-4\\1\\7\end{bmatrix}$ and $v_3 = \begin{bmatrix}-2\\1\\5\end{bmatrix}$. Do $\{v_1, v_2, v_3\}$ form a basis for $\mathbb{R}^3$

$a v_1 + b v_2 + c v_3 = 0$ for nonzero a, b, c ?

$$\left[\begin{array}{ccc|c} 3 & -4 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ -6 & 7 & 5 & 0 \end{array}\right] \xrightarrow{R_3+2R_1 \to R_3} \begin{bmatrix} 3 & -4 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3+R_2} \begin{bmatrix} 3 & -4 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

yes L.I, only solution is $a=b=c=0$

Spanning Set Theorem

Spanning Set Theorem (Chapter 4, Theorem 5)

Let $S = \{v_1, v_2, \ldots, v_p\}$ be a set of vectors in $\mathbb{R}^n$ and let $H = \text{Span}\,\{v_1, v_2, \ldots, v_p\}$.

1. If one of the vectors v_k is a linear combination of the remaining vectors, then the set formed from S by removing v_k still spans H.
2. If $H \neq \{0\}$, the some subset of S is a basis for H.

Row A

Example 4.3.2. Find a basis for Nul B and Col B where $B = \begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix}$

Row A

$$\begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix} \xrightarrow{RREF} \left[\begin{array}{ccccc|c} 1 & 4 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$x_1 + 4x_2 + 2x_4 = 0$

$x_3 - x_4 = 0$

$x_5 = 0$

$x_1 = -4x_2 - 2x_4$

$\cancel{x_2} = x_2$

$\cancel{x_3} = x_4$

$x_4 = x_4$

$x_5 = 0$

$$\text{nul } A = \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

x_2 $\qquad$ x_4