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Chapter 4 Examples , Linear Algebra 5e Lay

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4.3 Basis and linearly independent sets

Recall:

An indexed set of vectors $\{v_1, v_2, \dots, v_n\}$ is said to be **linearly independent** if the equation

$$a_1v_1 + a_2v_2 + \cdots + a_nv_n = 0$$

has only the trivial solution $a_1 = a_2 = \cdots = a_n = 0$. It is said to be **linearly dependent** otherwise.

Basi

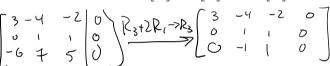
Definition 4.6. Let H be a vector space (including possibly that H is a subspace of some other vector space). A set of vectors

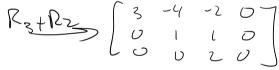
$$\mathcal{B} = \{b_1, b_2, \dots, b_n\}$$

is a **basis** for H if:

- 1. \mathcal{B} is a linearly independent set of vectors. (This means there is a pivot position in every column of the reduced matrix consisting of the columns of \mathcal{B}).
- 2. $H = \operatorname{Span} \mathcal{B}$.

Example 4.3.1. Let $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$ and $v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$. Do $\{v_1, v_2, v_3\}$ form a basis for \mathbb{R}^3





 $\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 0$

 $a \begin{bmatrix} 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $a = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

 $\begin{array}{c}
15 v_2 + CV_3 = 0 \\
\text{for nenzero} \\
a, b, c
\end{array}$

T, only solution is a=b=c=c

Spanning Set Theoren

Spanning Set Theorem (Chapter 4, Theorem 5)

Let $S = \{v_1, v_2, \dots, v_p\}$ be a set of vectors in \mathbb{R}^n and let $H = \mathrm{Span}\ \{v_1, v_2, \dots, v_p\}$.

- 1. If one of the vectors v_k is a linear combination of the remaining vectors, then the set formed from S by removing v_k still spans H.
- 2. If $H \neq \{0\}$, the some subset of S is a basis for H.

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