Chapter 2 Notes, Linear Algebra 6e Lay

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2.3 Characterizations of Invertible Matrices

Theorem 2.6. Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

- a. A is an invertible matrix.
- b. A is row equivalent to I_n
- c. A has n pivot positions.
- d. The equation Ax = 0 has only the trivial solution.

e. The columns of A form a linearly independent set.

- f. The linear transformation $x\mapsto Ax$ is one-to-one.
- From our knowledge as equations.

 homogeneous

 homogeneous

 homogeneous

 Solution

 Solution g. The equation Ax = b has at least one solution for each b in \mathbb{R}^n .
- h. The columns of A span \mathbb{R}^n .
- i. The linear transformation $x\mapsto Ax$ maps \mathbb{R}^n onto $\mathbb{R}^n.$
- j. There is an $n \times n$ matrix C such that CA = I. C = A
- k. There is an $n \times n$ matrix D such that AD = I.
- l. A^T is an invertible matrix.

Example 2.3.1. All matrices in this example are $n \times n$. Each part of the example is an implication of the form If "statement 1" then "statement 2". The implication is TRUE if "statement 2" is ALWAYS true whenever "statement 1" happens. If there is a time when it is not true then it is

1. If the equation Ax = 0 has only the trivial solution, then A is row equivalent to I_n .

2. If the columns of A span \mathbb{R}^n , then the columns are linearly independent.

3. If the equation Ax=0 has a nontrivial solution, then A has fewer than n pivot positions.

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5. If there is an $n \times n$ matrix D such that AD = I, then there is also and $n \times n$ matrix C such that CA = I.

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6. If the columns of A are linearly independent, then the columns of A span \mathbb{R}^n

7. If the equation Ax = b has a least one solution for each $b \in \mathbb{R}^n$, then the solution is unique for each b.

Since there is a solution for each b in IR" that A spans IR"

Theorem 2.7. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation and let A be the standard matrix for T. Then T is invertible if and only if A is an invertible matrix. In that case, the linear transformation S given by $S(x) = A^{-1}x$ is the unique function satisfying T(S(x)) = x and S(T(x)) = x.

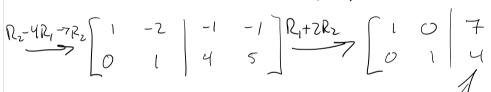
T(x) = A x

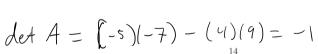
T'(x) = A'X

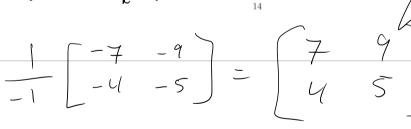
Example 2.3.2. T is a linear transformation from \mathbb{R}^2 into \mathbb{R}^2 . Show that T is invertible and find

a formula for
$$T^{-1}$$
.

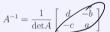
$$T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2) = \begin{pmatrix} -5x_1 + 9x_2 \\ 4x_1 - 7x_2 \end{pmatrix} = \begin{pmatrix} -5x_1 + 9x_2 \\ 4x_1 - 7x_2$$







t $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and define the determinant



atrix is not invertible