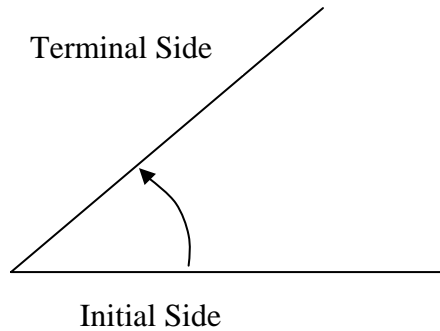## Section 9.1 Degrees and Radians:

An angle is the shape formed when two rays come together. In trigonometry we think on one of the sides as being the **Initial Side** and the angle is formed by the other side (**Terminal Side)** rotating away from the initial side.

We will usually draw our angles on the coordinate axes with the positive x – axis being the **Initial Side**. If we sweep out an angle in the counter clockwise direction we will say the angle is positive and if we sweep the angle in the clockwise direction we will say the angle is negative.
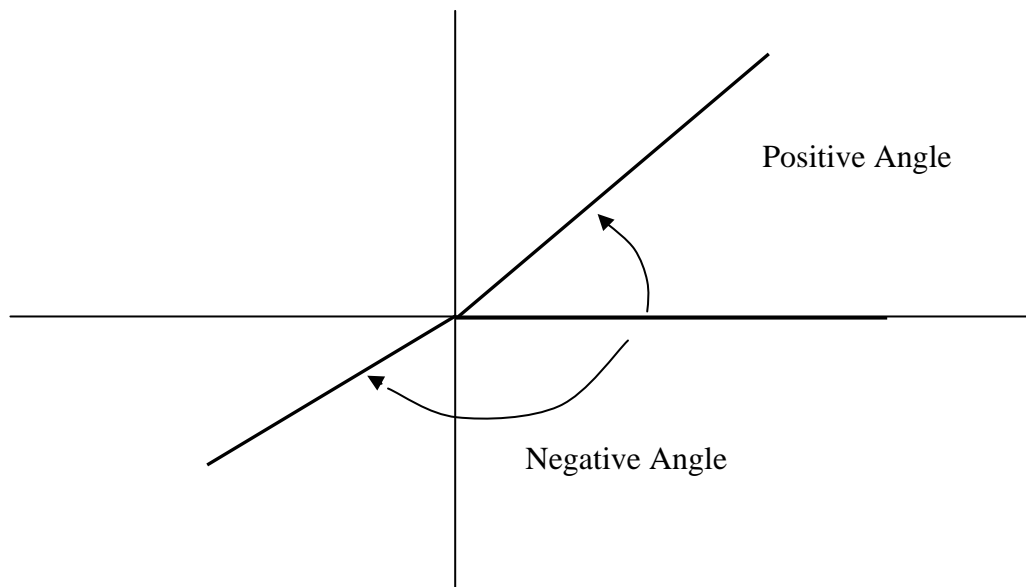
Measuring an Angle

When we measure angles we can think of them in terms of pieces of a circle. We have two units for measuring angles. Most people have heard of the **Degree** but the **Radian** is more useful in Trigonometry.

NOTE: By convention if the units are not specified they are **RADIANS**.

Degrees: one degree (1°) is a rotation of 1/360 of a complete revolution about the vertex. We can think of this as going all the way around the circle. There are 360 degrees in one full rotation.

Radian: One **Radian** is the measure of a central angle θ that intercepts an arc *s* equal in length to the radius *r* of the circle. Since the radian is measured in terms one *r* on the arc of a circle and the complete circumference of the circle is $2\pi r$ then there are 2π radians in one full rotation.
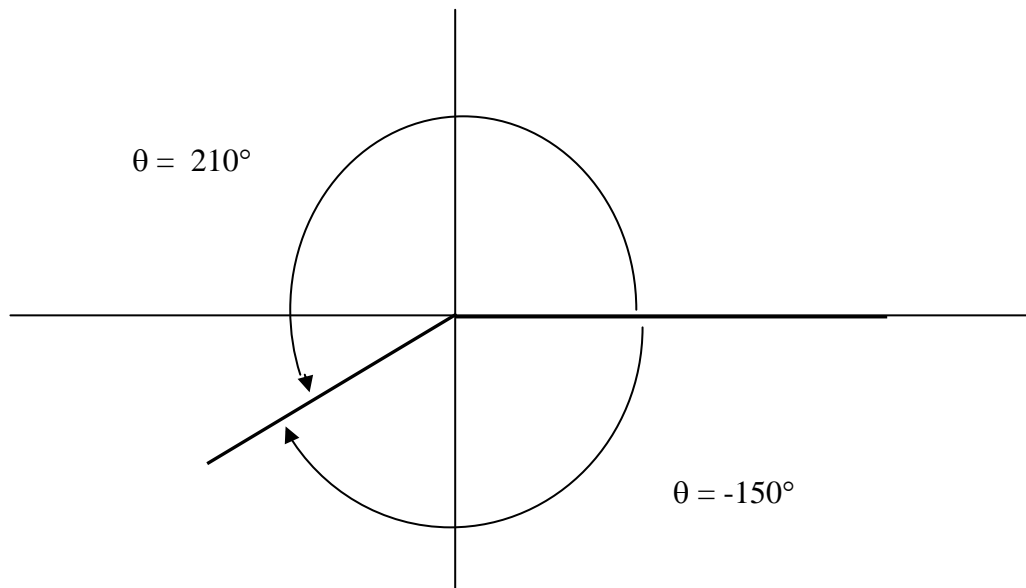
This gives us a way to convert between degrees and radians:

$360° = 2\pi$ rad

To convert from degrees → radians we multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$

To convert from radians → degrees we multiply radians by $\dfrac{180°}{\pi \text{ rad}}$

Consider the following two angles: 210° and -150°. If we sketch these two angles we will see that they look exactly the same:

θ = 210°

θ = -150°

Since these two angle terminate at the same place we call them **Coterminal Angles.** There are an infinite number of ways to draw an angle on the coordinate axes. By simply adding or subtracting 360° (or 2π rad) you will arrive at the same place.
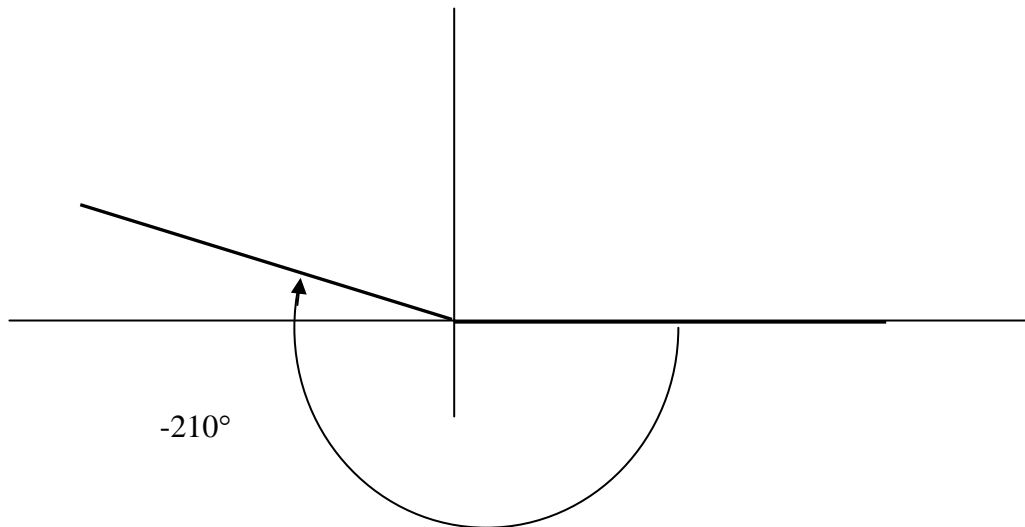
Ex 1: Convert the following to radians, sketch the angle and find two coterminal angles (one positive and one negative).

a) $30° = \left(30\deg\right)\left(\dfrac{\pi\,\text{rad}}{180\deg}\right) = \dfrac{\pi}{6}\,\text{rad}$

30°
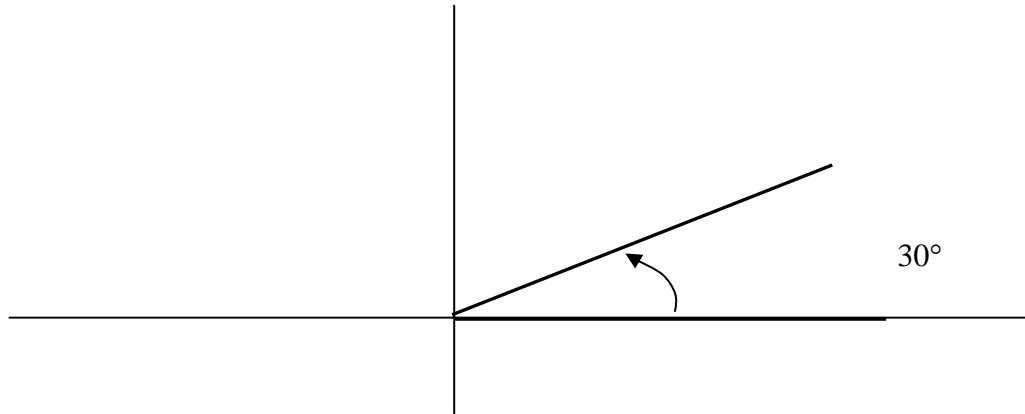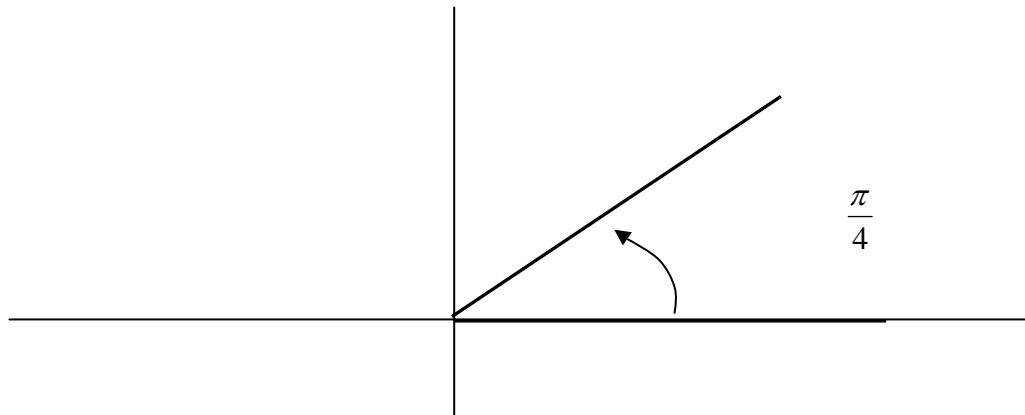
Coterminal angles: $30 + 360 = 390°$
$30 - 360 = -330°$

b) $-210° = \left(-210\deg\right)\left(\dfrac{\pi\,\text{rad}}{180\deg}\right) = -\dfrac{7\pi}{6}\,\text{rad}$

-210°

Coterminal angles: $-210 + 360 = 150°$
$-210 - 360 = -570°$

Ex 2: Convert the following to degrees, sketch the angle and find two coterminal angles (one positive and one negative).

a) $\quad \dfrac{\pi}{4}\,\text{rad} = \left(\dfrac{\pi}{4}\,\text{rad} \right)\!\left(\dfrac{180\,\text{deg}}{\pi\,\text{rad}} \right) = 45\,\text{deg}$
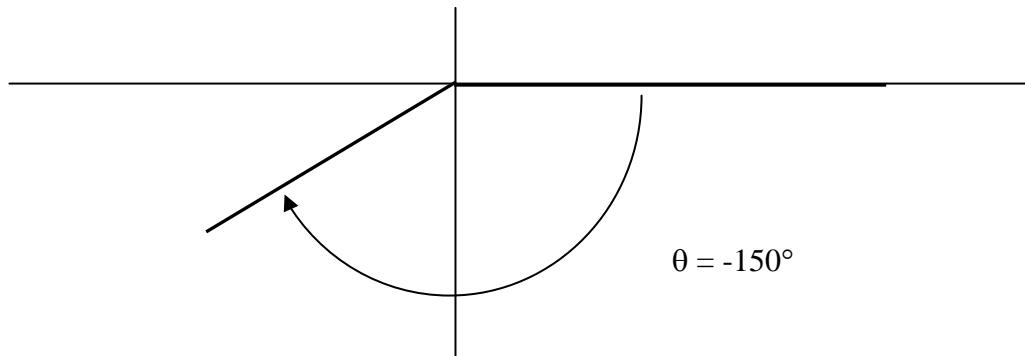

$$\dfrac{\pi}{4}$$

Coterminal angles: $\quad \dfrac{\pi}{4} + 2\pi = \dfrac{9\pi}{4}$

$$\dfrac{\pi}{4} - 2\pi = -\dfrac{7\pi}{4}$$

b) $\quad -\dfrac{5\pi}{6}\,\text{rad} = \left(-\dfrac{5\pi}{6}\,\text{rad} \right)\!\left(\dfrac{180\,\text{deg}}{\pi\,\text{rad}} \right) = -150\,\text{deg}$

$$\theta = \text{-}150°$$

Coterminal angles: $\quad -\dfrac{5\pi}{6} + 2\pi = \dfrac{7\pi}{6}$

$$-\dfrac{5\pi}{6} - 2\pi = -\dfrac{17\pi}{6}$$