Section 5.3: The First Derivative Test

Increasing and decreasing functions:
Definition: Let \( f \) be a function defined on an interval \( I \) and let \( x_1 \) and \( x_2 \) be any two point in \( I \).

1. \( f \) is increasing on \( I \) if \( x_1 < x_2 \), then \( f(x_1) < f(x_2) \)
2. \( f \) is decreasing on \( I \) if \( x_1 > x_2 \), then \( f(x_1) < f(x_2) \)

The first derivative test for increasing/decreasing.
Suppose that \( f \) is continuous on \([a, b]\) and differentiable on the open interval \((a, b)\).
If \( f'(x) > 0 \) for all \( x \) in \((a, b)\) then \( f \) increases on \([a, b]\).
If \( f'(x) < 0 \) for all \( x \) in \((a, b)\) then \( f \) decreases on \([a, b]\).

Local Extrema (Relative Extrema)
Definitions:
1. A function \( f \) has a **local maximum value** at a point \( c \) if it is the highest point near itself.
2. A function \( f \) has a **local minimum value** at a point \( c \) if it is the lowest point near itself.
3. A **critical number** of a function \( f \) is a number \( c \) in the domain of \( f \) such that \( f'(c) \) is zero or \( f'(c) \) does not exist.

The First Derivative Test for Local Extrema.
Let \( f \) be a continuous function on \([a, b]\) and \( c \) be a critical number in \([a, b]\).
1. If \( f''(x) \geq 0 \) on \((a, c)\) and \( f''(x) \leq 0 \) on \((c, b)\), then \( f \) has a local maximum of \( f(c) \) at \( x = c \).
2. If \( f''(x) \leq 0 \) on \((a, c)\) and \( f''(x) \geq 0 \) on \((c, b)\), then \( f \) has a local minimum of \( f(c) \) at \( x = c \).
3. If \( f'' \) does not change signs at \( x = c \), then \( f \) has no local extrema at \( x = c \).

Ex 1: Find the relative extrema for the function below.
Ex 2: Find the relative extrema for \( f(x) = -2x^2 + 4x + 3 \)

**Step 1:** Find the critical numbers: Set \( f'(x) = 0 \) and find where \( f'(x) \) does not exist

**Step 2:** Make a table

Ex 3: Find the relative extrema for \( f(x) = x^2(3 - x) \)
Ex 4: Find the relative extrema for \( f(x) = (x - 1)^{1/3} \)

Ex 5: Find the relative extrema for \( f(x) = x + \frac{1}{x} \)