Mth 176 - Review Problems for Test 1

For Test #1 study these problems, the examples in your notes, and the homework.

I  Evaluate the following:

1. \[ \int \frac{x^3}{\sqrt{1-x^4}} \, dx \]
2. \[ \int \frac{x}{\sqrt{1-x^4}} \, dx \]
3. \[ \int \frac{\ln(x)^2}{x} \, dx \]
4. \[ \int_{\pi/2}^{\pi/2} \frac{\sin x}{\cos x + 1} \, dx \]
5. \[ \int_{\pi/2}^{\pi/2} \frac{\sin x}{(\cos x + 1)^2} \, dx \]
6. \[ \int_{\pi/2}^{\pi/2} \frac{\sin x}{\cos^2 x + 1} \, dx \]
7. \[ \int_{1/6}^{1/6} \frac{dx}{\sqrt{1-9x^2}} \]
8. \[ \int_{-1}^{3} \frac{x}{\sqrt{1+x}} \, dx \]
9. \[ \int_{3/4}^{3/2} \frac{3x^2 - 4x + 2}{x} \, dx \]
10. \[ \int_1^2 \frac{\sec (3x) \tan (3x)}{(1 + \sec (3x))^3} \, dx \]
11. \[ \int_0^{\ln^2} e^{5x} \, dx \]
12. \[ \int_0^{\pi^2/9} \sec^2 \sqrt{x} \, dx \]
13. \[ \int (2 - e^{3x})^2 \, dx \]
14. \[ \int 2x^4 \, dx \]
15. \[ \int \frac{e^{3x} - e^{2x}}{e^x - 1} \, dx \]
16. \[ \int_0^1 e^x \, dx \]
17. \[ \int_0^3 e^x \frac{\sin x}{\cos x} \, dx \]
18. \[ \int_0^1 (x^7 + 7x + 7^x) \, dx \]
19. \[ \int_{-2}^{3} |x| \, dx \]
20. \[ \int_0^4 x - \sqrt{16 - x^2} \, dx \]
21. \[ \int \frac{x}{x^3 + 3} \, dx \]
22. \[ \int_0^2 (x - 2)^5 \, dx \]
23. \[ \int_0^1 xe^{x^2} \, dx \]
24. \[ \int_0^1 \frac{x}{(e^x + 1)^5} \, dx \]
25. \[ \int_0^2 x^2 (x - 2)^10 \, dx \]
26. \[ \int_0^{\pi/6} \cos x \sin^5 x \, dx \]
27. \[ \int_0^\pi \frac{\sin x}{x^4 + x^2 + 1} \, dx \]

II  Solve the following:

28. Determine a function \( y = f(x) \) such that:
   (a) \( \frac{dy}{dx} = e^{-x} \) and \( y(0) = 1 \)
   (b) \( \frac{dy}{dx} = \frac{1}{x^2} \) and \( y(1) = 1, \ x > 0 \)
   (c) Let \( f'''(x) = 2 \sin x + e^x, \ f''(\frac{\pi}{2}) = 0, \ f'(0) = 2, \ f(0) = 1 \)

29. Evaluate each of the following or conclude that the solution cannot be determined from the information given.
   (a) If \( \int_2^8 f(x) \, dx = 1.7 \) and \( \int_5^8 f(x) \, dx = 2.5 \), find \( \int_2^5 f(x) \, dx \)
   (b) If \( \int_0^4 f(x) \, dx = 5 \) and \( \int_0^4 g(x) \, dx = 2 \), find \( \int_0^4 f(x) \cdot g(x) \, dx \)

30. Use the definition of the definite integral to evaluate \( \int_1^3 (x^2 - x) \, dx \).
31. Use the max-min inequality (property 8, pg 375) to find upper and lower boundaries for 
\[ \int_1^3 \sqrt{x^2 + 3} \, dx. \]

32. Use the properties of integrals to verify 
\[ \int_0^1 e^x \cos x \, dx \leq e - 1. \]

33. Find the area of the region bounded by the \( y \)-axis, the line \( y = 1 \) and the curve \( y = \sqrt[4]{x} \) by writing \( x \) as a function of \( y \) and integrating with respect to \( y \).

34. Given the function \( y = x \) on the interval \([-2, 2]\),

(a) Find the area of the region bounded by the \( x \)-axis and the function \( y = x \) from \( x = -2 \) to \( x = 2 \).

(b) Evaluate the integral \( \int_{-2}^{2} x \, dx \).

35. Use the Midpoint Rule with \( n = 5 \) to approximate 
\[ \int_2^7 (1 + x^2) \, dx. \]

36. Let \( f(x) = 2x^2 - 1 \) defined on the interval \([1, 3]\). Find the Riemann sum of \( f \) given the partition defined by \( 1, \frac{3}{2}, \frac{9}{4}, \frac{11}{4}, 3 \) and where \( x^*_1 = 1, x^*_2 = \frac{7}{4}, x^*_3 = \frac{11}{4}, x^*_4 = 3 \).

37. Evaluate each of the following:

(a) Let \( f(x) = \int_{x^2}^{0} t^4 \, dt \). Find \( f''(x) \).

(b) Let \( f(x) = \int_{0}^{\sqrt{x}} \sqrt{t^4 + 20} \, dt \). Find \( f'(4) \).

38. On the surface of the moon, a ball is thrown directly upward at a speed of 64 feet per second from a cliff 80 feet above the ground. The acceleration due to lunar gravity is 5.2 \( \text{ft/sec}^2 \).

(a) Find the expressions for the velocity and the height of the ball \( t \) seconds after it was released.

(b) At what time does the ball reach its highest point? How high above the ground at the base of the cliff does it reach?

(c) When does the ball strike the ground at the base of the cliff? What is its velocity at that instant?
39. Let \( f(x) = x^2 \) on the interval \([0, 2]\). Let the interval be divided into four equal subintervals. Find the value of the Riemann sum approximation if each \( x_i^* \)

(a) is the left-endpoint of the subinterval; and

(b) is the right-endpoint of the subinterval.

40. A particle moves on an oriented axis with velocity given by \( v(t) = 2t - 3, \ t \geq 0 \). It is known that for \( t = 0 \) the particle is in the position \( x = 5 \). Find the position function \( x = x(t) \).

41. Sam calculated the value of \( \int_{-1}^{1} (1 + x^2)^2 \, dx \) using the following argument:

Make the substitution \( u = 1 + x^2 \). The new limits of integration would both equal 2 because \( x = -1 \rightarrow u = 1 + (-1)^2 = 2 \) and \( x = 1 \rightarrow u = 1 + (1)^2 = 2 \). Since the new limits of integration are equal, this means that \( \int_{-1}^{1} (1 + x^2)^2 \, dx = 0 \). Where is the mistake in Sam’s argument?

42. The following graph of \( f \) consists of line segments and semicircles. Use it to evaluate the requested integrals.

\[
\begin{align*}
\text{(a)} & \quad \int_{0}^{14} f(x) \, dx \\
\text{(b)} & \quad \int_{0}^{10} f(x) \, dx \\
\text{(c)} & \quad \int_{3}^{12} f(x) \, dx
\end{align*}
\]

43. Express the following limits as definite integrals and evaluate:

\[
\begin{align*}
\text{(a)} & \quad \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n (3 + \frac{2i}{n})} \\
\text{(b)} & \quad \lim_{n \to \infty} \sum_{i=1}^{n} i^3 \\
\text{(c)} & \quad \lim_{n \to \infty} \sum_{i=1}^{n} e^{1+\left(\frac{5i}{n}\right)} \left( \frac{5}{n} \right) \\
\text{(d)} & \quad \lim_{n \to \infty} \frac{1}{n} \left( \frac{1}{1 + \left( \frac{1}{n} \right)^2} + \frac{1}{1 + \left( \frac{2}{n} \right)^2} + \frac{1}{1 + \left( \frac{3}{n} \right)^2} + \cdots + \frac{1}{1 + \left( \frac{n}{n} \right)^2} \right)
\end{align*}
\]