Math 176 Calculus – Sec. 6.5: Average Value of a Function

I. Average Value of a Function
A. Defn: If \( f \) is integrable on \([a, b]\), its average (mean) value on \([a, b]\) is

\[
\bar{f}_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx
\]

B. EXAMPLE
Find the average value of \( f(x) = \sqrt{x} \) on \([0, 4]\).

II. Mean Value Theorem for Integral Calculus
A. The Mean Value Theorem for Definite Integrals (MVTh\textsuperscript{m}): If \( f \) is continuous on \([a, b]\), then at some point \( c \) in \([a, b]\),

\[
f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx
\]

B. EXAMPLE
Apply MVTh\textsuperscript{m} to the example above.

C. Geometric Interpretation of the MVTh\textsuperscript{m}
For positive functions \( f \), there is a number \( c \) such that the rectangle with base \([a, b]\) and height \( f(c) \) has the same area as the region under the graph of \( f \) from \( a \) to \( b \).
III. Additional Examples

1. Find the average value of \( y = \frac{x}{\sqrt{9 + x^2}} \) on \([0,4]\). Apply MVTh to this problem.

2. In a certain city the temperature (in °F) \( t \) hours after 9 a.m. was approximated by the function \( T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right) \). Find the average temperature during the period from 9 a.m. to 9 p.m.