8.1 Systems of linear equations: Gaussian Elimination

everything to the 1st power.

What does it mean to be a **solution** to a system of equations?

- It is the set of all ordered pairs (x, y) that satisfy the two equations. You can have one solution, multiple solutions, infinite solutions or no solutions.

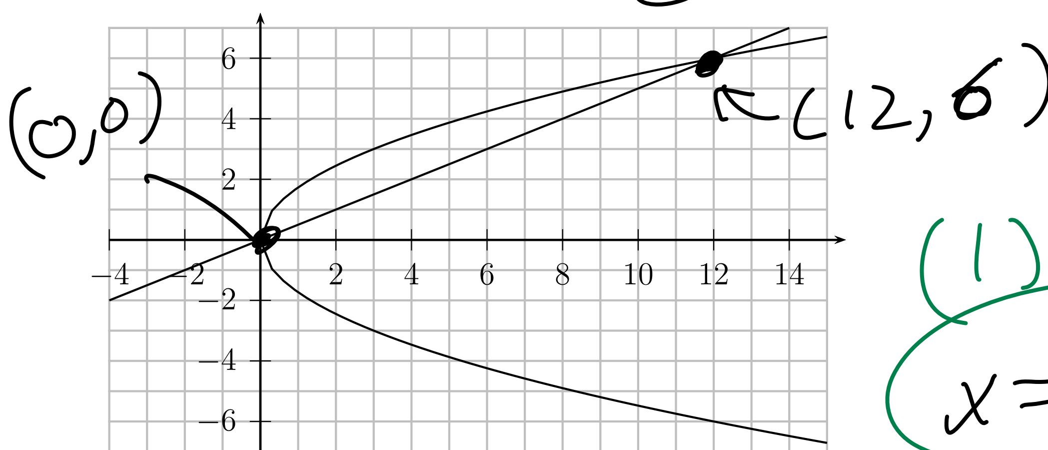
Three methods of solution

I. Graphing: Graph the two equations and find the intersection.

Example 8.1.1. Solve the following system of equations by graphing.

$$\begin{cases} x - 2y = 0 & (1) \\ 3x - y^2 = 0 & (2) \end{cases}$$

Not linear



II. Substitution

Solve one of the equations for x or y and substitute into the other equation.

(1)

$x = 2y$

$y = \frac{1}{2}x$

Example 8.1.2. Use substitution to solve the following system of equations.

$$\begin{cases} x - 2y = 0 & (1) \\ 3x - y^2 = 0 & (2) \end{cases}$$

In this case we solve equation (1) for x and substitute into equation (2).

(2)

$3(2y) - y^2 = 0$

$y(6 - y) = 0$

$\boxed{\begin{matrix} y = 0 \\ x = 0 \end{matrix}}$ or $\boxed{\begin{matrix} y = 6 \\ x = 12 \end{matrix}}$

$x = 2y$

$x = 2(0) = 0$

OR $x = 2(6) = 12$

$(12, 6)$

III. Elimination (addition method)

Multiply the equations by appropriate constants so that, when you add the two equations, one variable cancels.

$2 + 3x - 5y = 10 + 2$

Example 8.1.3. Solve by elimination.

$$(-3)\begin{cases} (x + 7y) = 12\ (-3) \\ 3x - 5y = 10 \end{cases} \text{ multiply (1) by } -3$$

$$\begin{aligned} &\boxed{-3x} - 21y = -36 \\ + &\boxed{3x} - 5y = 10 \\ \hline &0x - 26y = -26 \end{aligned} \text{ add the equations}$$

$y = 1$

$x + 7(1) = 12$

$x = 5$

$$\begin{cases} 5\,(x + 7y = 12) \\ 7\,(3x - 5y = 10) \end{cases}$$

$$\begin{aligned} &5x \boxed{+35y} = 60 \\ + &21x \boxed{-35y} = 70 \\ \hline &26x = 130 \end{aligned}$$

$x = 5,\ y = 1$

Example 8.1.4. Solve by any method.

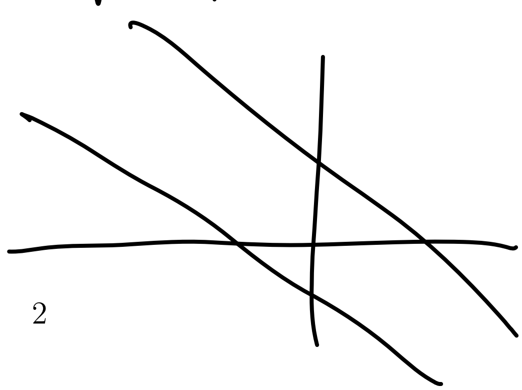
$$6\begin{cases} 6x + 5y = -3 \\ (-x - \frac{5}{6}y) = (-7)\ 6 \end{cases}$$

$$\begin{aligned} &6x + 5y = -3 \\ + &-6x - 5y = -42 \\ \hline &0 = -45 \end{aligned}$$

Parallel lines

Not true

No solution



Example 8.1.5. Solve by any method.

$$3\left\{\begin{array}{rcl}\left(-\frac{2}{3}x+y\right) & = & (-2)\,3 \\ 2x-3y & = & 6\end{array}\right.$$

$$\begin{array}{r}-2x+3y=-6 \\ +\quad 2x-3y=6 \\ \hline 0=0\end{array}$$

$\boxed{\text{Infinite solutions — the same line}}$

Example 8.1.6. Solve by any method.

$x=3$

$$\left\{\begin{array}{rcl}2x+3y & = & 18 \\ (5x-y & = & 11)(3)\end{array}\right. \leftarrow$$

$$\begin{array}{r}2x+3y=18 \\ 15x-3y=33 \\ \hline 17x=51\end{array}$$

$x=3$

$5(3)-y=11$

$y=15-11=4$

$\boxed{(3,4)}$

Systems of equations

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Solve the system of equations:

$$\begin{cases} 3x - 8y + z = 34 \\ -x + y - z = -7 \\ x - 3y = 12 \end{cases}$$

write first because would like 1x in first equation.

$$\begin{aligned} (1)\quad & x - 3y = 12 \\ (2)\quad & 3x - 8y + z = 34 \\ (3)\quad & -x + y - z = -7 \end{aligned}$$

eliminate the x's in eq (2) & (3)

eliminate x's first

$$\begin{aligned} (1)\quad & \quad\; -3y = 12 \\ (3)\quad & + \; -x + y - z = -7 \\ \hline & -2y - z = 5 \end{aligned}$$

New equation 3

$$\begin{aligned} (1)\quad & (x - 3y = 12)(-3) \\ (2)\quad & 3x - 8y + z = 34 \\ (3)\quad & 0x - 2y - z = 5 \end{aligned}$$

$-3(1) + (2)$

$$\begin{aligned} -3(1)\quad & -3x + 9y = -36 \\ +\;(2)\quad & 3x - 8y + z = 34 \\ \hline & y + z = -2 \end{aligned}$$

New equation 2

$$\begin{aligned} (1)\quad & x - 3y = 12 \\ (2)\quad & y + z = -2 \\ (3)\quad & -2y - z = 5 \end{aligned}$$

2 equations 2 unknowns

(2) $y + z = -2$
(3) $-2y - z = 5$

2 unknowns

(2) $y + z = -2$

$+$ (3) $+ \; -2y - z = 5$

$-y = 3$

$y = -3$

Back substitution

put $y = -3$ into the other equations

$-3 + z = -2$

$z = 1$

$x - 3(-3) = 12$

$x + 9 = 12$

$x = 3$

$$\boxed{\text{answer: } (3, -3, 1) = (x, y, z)}$$

Question 8 0/1 pt 10 99 Details

The admission fee at an amusement park is $1.50 for children and $4 for adults. On a certain day, 292 people entered the park, and the admission fees collected totaled 888.00 dollars. How many children and how many adults were admitted?

Your answer is

number of children equals

number of adults equals

8.6 Partial Fraction Decomposition

multiply

Partial Fractions consists of decomposing a rational function into simpler component fractions and then evaluating the integral term by term.

Example 8.6.1. Denominator is a product of disctinct linear factors

$$\frac{3x+7}{x^2+6x+5} = \left[\frac{3x+7}{(x+1)(x+5)} = \frac{\cancel{A}\ 1}{x+1} + \frac{\cancel{B}\ 2}{x+5}\right] (x+1)(x+5)$$

$$\frac{(3x+7)\cancel{(x+1)}\cancel{(x+5)}}{\cancel{(x+1)}\cancel{(x+5)}\ 1} = \frac{A\cancel{(x+1)}(x+5)}{\cancel{(x+1)}\ 1} + \frac{B(x+1)\cancel{(x+5)}}{1\ \cancel{(x+5)}}$$

short cut starts here

$$3x+7 = A(x+5) + B(x+1)$$

$$3x+7 = Ax+5A+Bx+B$$

$$3x+7 = (A+B)x + (5A+B)$$

$$(-1)(A+B=3)$$
$$+\ 5A+B=7$$
$$4A = 4$$

$$\boxed{A=1,\ B=2}$$

Example 8.6.2. Denominator is a product of linear factors, some of which are repeated.

$$\left[\frac{3x^2-8x+13}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}\right] (x+3)(x-1)^2$$

$$3x^2-8x+13 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)$$

$x=1$: $3-8+13 = 0A+0B+4C$

$8 = 4C \Rightarrow C=2$

$x=-3$: $3(9)+24+13 = (-3-1)^2 A + 0B + 0C$

$64 = 16A$

$4 = A$

$B = -1$

$x=0$: $13 = 4(-1)^2 + B(3)(-1) + 2(3)$

Example 8.6.3. Denominator contains irreducible quadratic factors, none of which is ~~repeated~~.

$$\frac{2x^2+x-8}{x^3+4x} = \frac{2x^2+x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} \quad x(x^2+4)$$

one degree lower than denominator.

$$\boxed{= \frac{-2}{x} + \frac{4x+1}{x^2+4}}$$

$$2x^2+x-8 = \overset{-2}{\cancel{A}}(x^2+4) + (Bx+C)x$$

$x=0$ $\quad -8 = 4A$

$-2 = A$

$$2x^2+x-8 = -2x^2-8+Bx^2+Cx$$

$$2x^2 + x - 8 = (-2+B)x^2 + Cx - 8$$

$Cx = 1x$

$C = 1$

$-8 = -8$

$$2x^2 = (-2+B)x^2$$

$-2+B=2$

$B = 4$