5.1 Function Composition

The **composition** of a function f with a function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Example 5.1.1. Suppose $f(x) = x^3 + 2x + 1$ and $g(x) = x - 1$ then find

a) $(f \circ g)(x) = f(g(x)) = f(x - 1)$

b) $(g \circ f)(x) = g(f(x)) = g(x^3 + 2x + 1)$

c) $(f \circ f)(x) = f(f(x)) = f(x^3 + 2x + 1)$

Example 5.1.2. Find (a) $(f \circ g)(x)$, (b) $(g \circ f)(x)$ and (c) the domain of each for $f(x) = \sqrt{x-4}$ and $g(x) = x^2$.

Example 5.1.3. Find (a) $(f \circ g)(x)$, (b) $(g \circ f)(x)$ and (c) the domain of each for

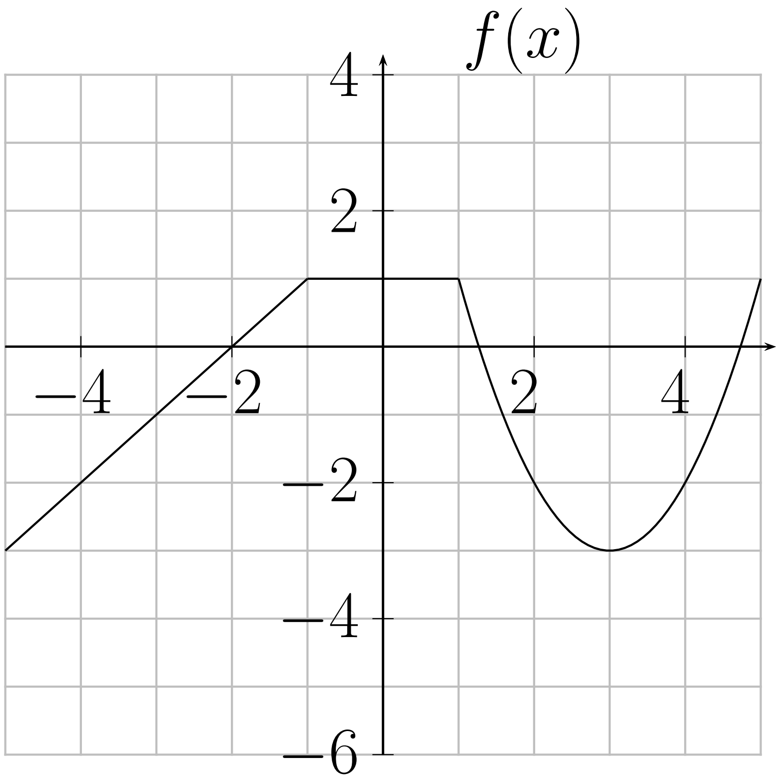
$$f(x) = \frac{1}{x-4} \quad \text{and} \quad g(x) = \frac{2}{x} + 2$$

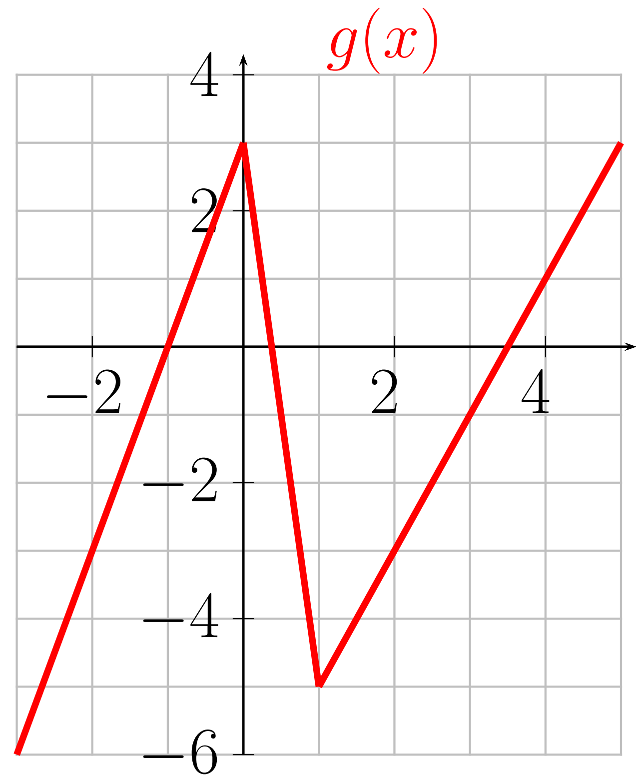
Example 5.1.4. Find functions f and g such that

$$h(x) = \sqrt[3]{x^2 - 4} = (f \circ g)(x)$$

Example 5.1.5. Let $f(x) = \sqrt{90 - x}$ and $g(x) = x^2 - x$. Find $(f \circ g)(x)$ and its domain.

Example 5.1.6. Use the functions below to find the compositions.





$f(g(1)) =$

$g(f(2)) =$

$f(g(0)) =$

$g(f(0)) =$

$f(g(-2)) =$

$g(f(-2)) =$

5.2 Inverse Functions

A function is said to be **one to one (1 - 1)** if no two ordered pairs have the same second component but different first component.

A function has one y value for each x value but those y values can repeat. In a 1 - 1 function the y values never repeat.

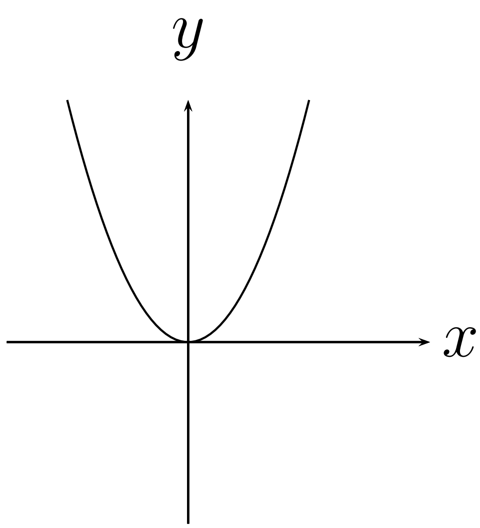
Graphically:

An equation must pass the Vertical Line Test to be a function.

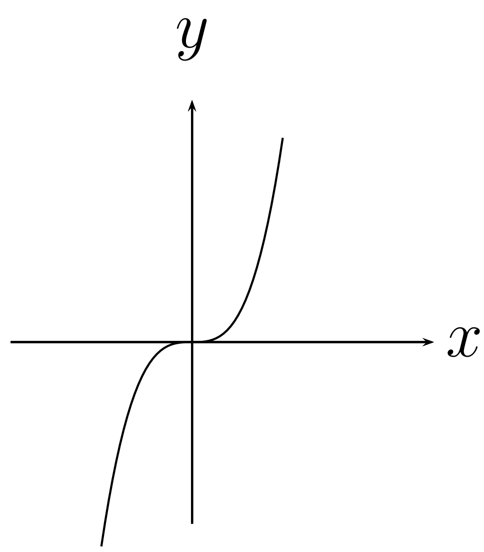
A function must pass the Horizontal Line Test to be 1 - 1.

Example 5.2.1. Use the horizontal line test on these graphs.

$f(x) = x^2$ is not 1 - 1.



$f(x) = x^3$ is 1 - 1.



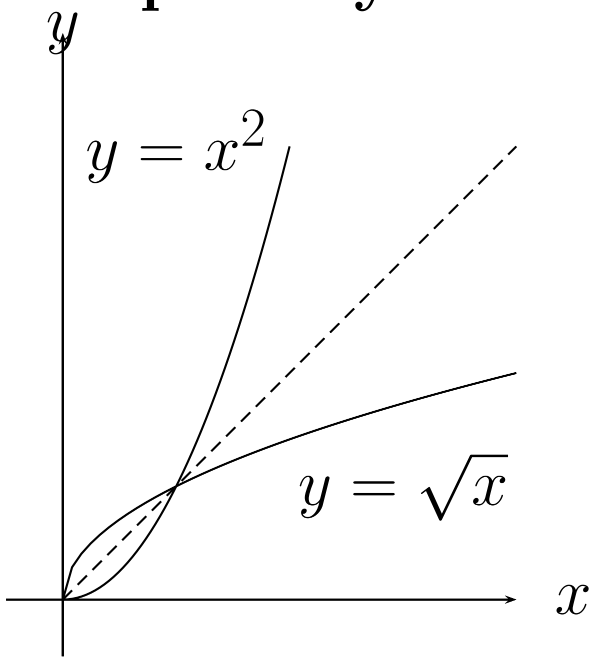
Inverses

The **indentity function** is $f(x) = x$ or $y = x$. You get out what you put in. Given a function f that is 1 - 1 then f has an inverse f^{-1} and

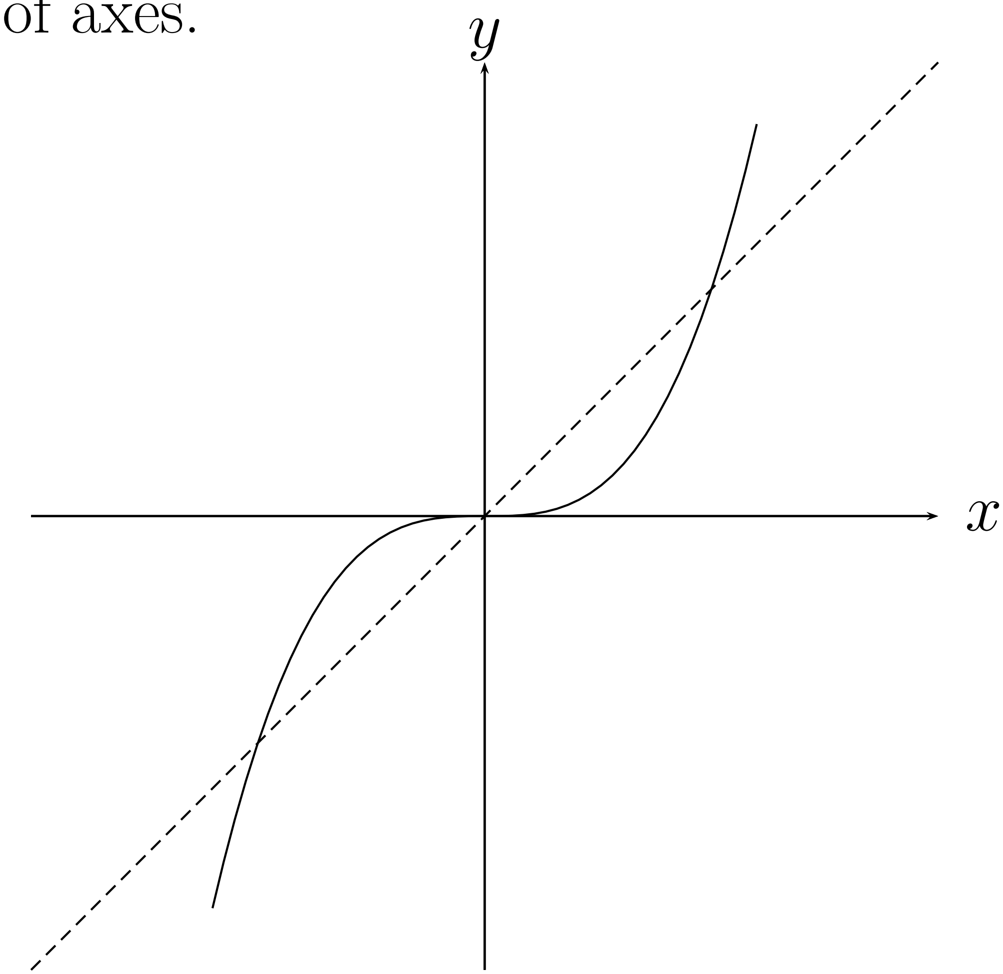
$$f(f^{-1}(x)) = x \qquad \text{and} \qquad f^{-1}(f(x)) = x.$$

If f is not 1 - 1 then f^{-1} DOES NOT EXIST.

Graphically



Example 5.2.2. Draw the inverse of the following function on the same set of axes.



5.2.1 Finding inverses from tables and graphs

Example 5.2.3. Use the table below to fill in the missing values.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	8	4	5	7	2	1	6	9	3	0

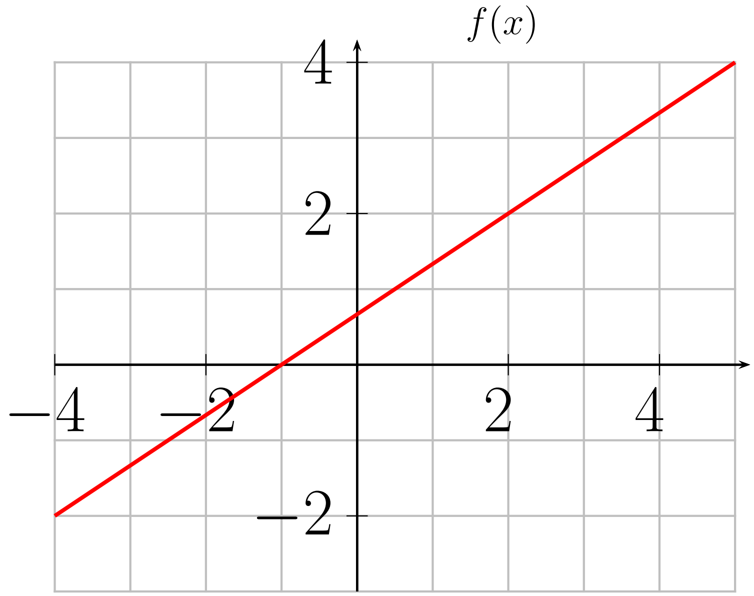
1. $f(2) =$

2. if $f(x) = 4$ then $x =$

3. $f^{-1}(5) =$

4. if $f^{-1}(x) = 1$ then $x =$

Example 5.2.4. Use the graph below to fill in the missing values.



$f(2) =$

if $f(x) = 4$ then $x =$

$f^{-1}(-2) =$

$f^{-1}(0) =$

if $f^{-1}(x) = 5$ then $x =$

5.2.2 Finding Inverses Algebraically

Step 1: Solve for x.
Step 2: Check the domain.
Step 3: Switch x and y.
Step 4: Write $f^{-1}(x) =$
Step 5: Check that $f(f^{-1}(x)) = x$.

Example 5.2.5. Let $f(x) = (x + 4)^2$.

1. Find the domain on which f is one-to-one and non-decreasing.

2. Find the inverse of $f(x) = (x+4)^2$ restricted to this domain.

Step 1: Solve for x. (two answers here)

Step 2: Check the domain.

Step 3: Switch x and y.

Step 4: Write $f^{-1}(x) =$

Step 5: Check that $f(f^{-1}(x)) = x$.

Example 5.2.6. Find the inverse of $f(x) = \dfrac{2x - 5}{-4x - 2}$.