4.1 Introduction to Rational Functions

 $\mathbf{Q}:$ What is a rational function?

A: It is a function of the form:

$$f(x) = \frac{N(x)}{D(x)} \qquad D(x) \neq 0.$$

Vertical and Horizontal Asymptotes

A Vertical Asymptote describes the behavior of a function near a discontinuity. They occur at any x - value where the numerator IS NOT equal to zero but the denominator IS equal to zero. **Example 4.1.1.** Find the domain and vertical asymptotes for

$$f(x) = \frac{1}{x}$$
 and $f(x) = \frac{1}{x-3}$ and $f(x) = \frac{1-3x}{x^2+12x+32}$.

A Horizontal Asymptote describes the behavior of a function as x gets very large. (ie. What happens to y as x goes to ∞ ?)

Horizontal Asymptotes

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}$$

where N(x) and D(x) have no common factors. The graph of f has one or no **horizontal asymptote** determined by comparing the degrees of n(x) and D(x).

- 1. If n < m, then the graph of f has the line y = 0 (the x-axis) as a horizontal asymptote.
- 2. If n = m then the graph of f has the line $y = \frac{a_n}{b_m}$ as a horzontal asymptote.
- 3. if n > m then the graph of f has no horizontal asymptote.

Example 4.1.2. Find the domain of the function and identify any horizontal and vertical asymptotes. Sketch a graph for each.

1.
$$f(x) = \frac{x-4}{(x-2)^2}$$

2.
$$f(x) = \frac{x-4}{1+2x}$$

3.
$$f(x) = \frac{(x-4)^2}{(x+1)^2}$$

4.
$$f(x) = \frac{(x-4)^2}{x-3}$$

5.
$$f(x) = \frac{1}{x} + 2$$

4.2 Graphing Rational Functions

Example 4.2.1. Let

$$f(x) = \frac{3x^2 + 4x + 1}{3x^2 + 11x - 20}$$

sketch the graph with the

- 1. y-intercept(s)
- 2. x-intercept(s)
- 3. vertical asymptote(s)
- 4. horizontal asymptote(s)

Example 4.2.2. Sketch and write an equation for a rational function with:

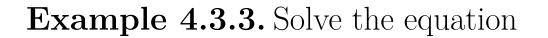
- 1. Vertical asymptotes at x = 5 and x = -5
- 2. *x*-intercepts at x = 1 and x = 2
- 3. y intercept at 3

4.3 Rational Equations and Applications

Example 4.3.1. Suppose f varies inversely with g and that f = 36 when g = 6. What is the value of f when g = 12?

Example 4.3.2. Solve the equation

$$\frac{2}{x} = \frac{4}{3x} - 5.$$



$$\frac{8}{x+1} - \frac{5}{2} = \frac{4}{3x+3}.$$

Example 4.3.4. Solve the equation

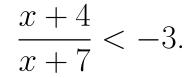
$$\frac{x}{2x-4} - 9 = \frac{1}{x-2}.$$

Example 4.3.5. Solve the equation

$$\frac{x+1}{x-1} = \frac{-1}{x+3} + \frac{8}{x^2+2x-3}$$

4.3.1**Rational Inequalities**

Example 4.3.6. Solve the inequality $\frac{x+4}{x+7} < -3.$



Example 4.3.7. Solve the inequality

$$\frac{x-2}{x^2-25} < -3.$$