Contents

1.1 Sets of Real Numbers and the Cartesian Coordinate Plane

Sets and Interval Notation

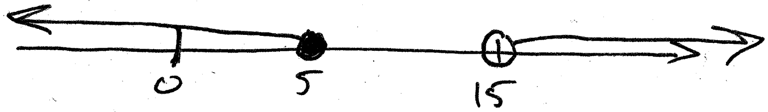
Definition 1.1. Suppose A and B are two sets.

- The **intersection** of A and B: $A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}$
- The **union** of A and B: $A \cup B = \{x \,|\, x \in A \text{ or } x \in B \text{ (or both)}\}$

set $\{x \leq 5$ or $x \geq 15\}$

Interval notation

$(-\infty, 5] \cup (15, \infty)$

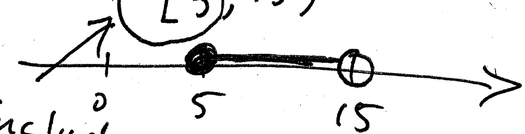


$\{x \geq 5$ and $x < 15\}$

$5 \leq x < 15$ Best

$[5, \infty) \cap (-\infty, 15)$

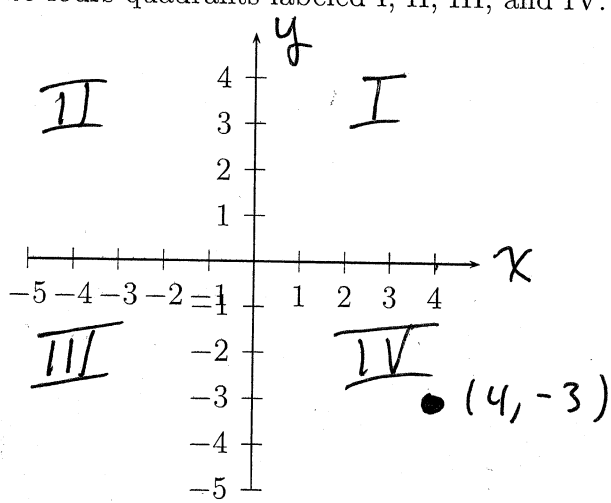
$[5, 15)$



Include 5

Cartesian Coordinates

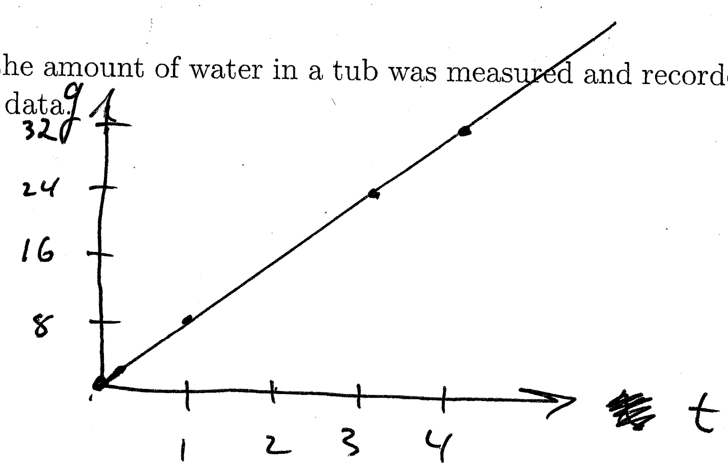
All points in the plane are ordered pairs (x, y) where the 1st coordinate is directed distance on the x - axis and the 2nd coordinate is directed distance on the y - axis. The xy-plane is divided into fours quadrants labeled I, II, III, and IV.



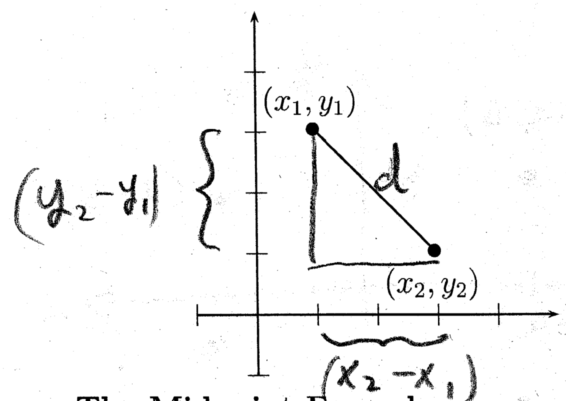
$(4, -3) = (x, y)$

Example 1.1.1. At various times, the amount of water in a tub was measured and recorded in the table of values. Sketch a plot of the data.

Time (min)	Water in tub (gallons)
0	0
1	8
3	24
4	32



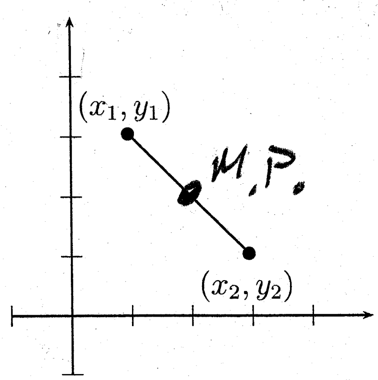
The distance Formula

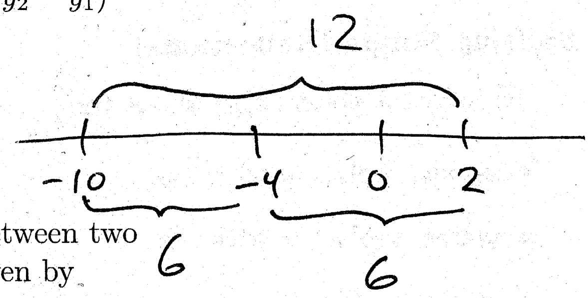


The distance between two points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula





The midpoint between two points is given by

$$\text{M.P.} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

$(x_1, y_1) \quad (x_2, y_2)$

Example 1.1.2. Find the distance and midpoint between $P(3, -10)$ and $Q(-1, 2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1-3)^2 + (2-(-10))^2}$$

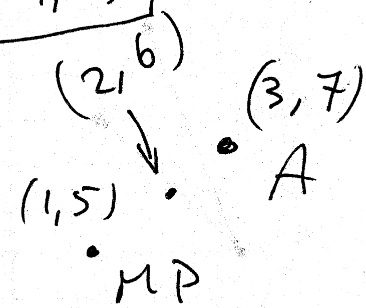
$$= \sqrt{16 + 144} = \sqrt{160} = \sqrt{16 \cdot 10}$$

$$\boxed{= 4 \cdot \sqrt{10}}$$

$$\text{M.P.} = \left(\frac{3 + (-1)}{2}, \frac{-10 + 2}{2}\right) \boxed{= (1, -4)}$$

Example 1.1.3. The midpoint of AB is at $(1, 5)$. If $A = (3, 7)$, find B. $\boxed{(-1, 3)}$

(x_1, y_1)



$$MP = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

$$(1, 5) = \left(\frac{x_2 + 3}{2}, \frac{y_2 + 7}{2}\right)$$

$$2(1) = \left(\frac{x_2 + 3}{2}\right)2 \qquad \left(5 = \frac{y_2 + 7}{2}\right)2$$

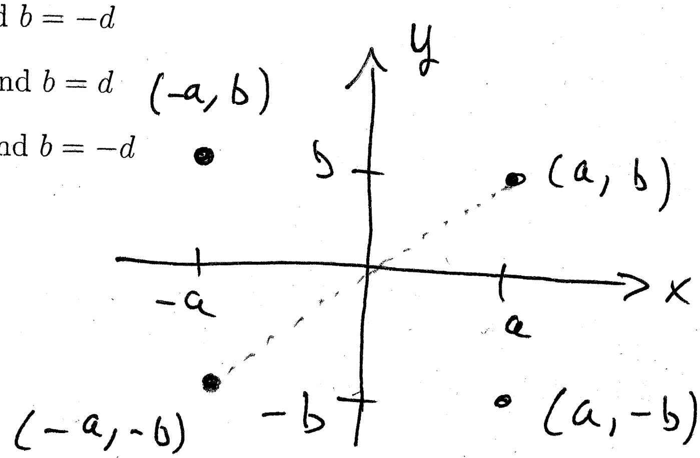
$$2 = x + 3 \qquad 10 = y + 7$$

$$\boxed{-1 = x} \qquad \boxed{3 = y}$$

$B\ (x_2, y_2)$

Definition 1.2. Two points (a, b) and (c, d) in the plane are said to be

* • **symmetric about the x-axis** if $a = c$ and $b = -d$
- **symmetric about the y-axis** if $a = -c$ and $b = d$
- **symmetric about the origin** if $a = -c$ and $b = -d$



Shifting Points (Reflections)

To reflect a point (x, y) about the:

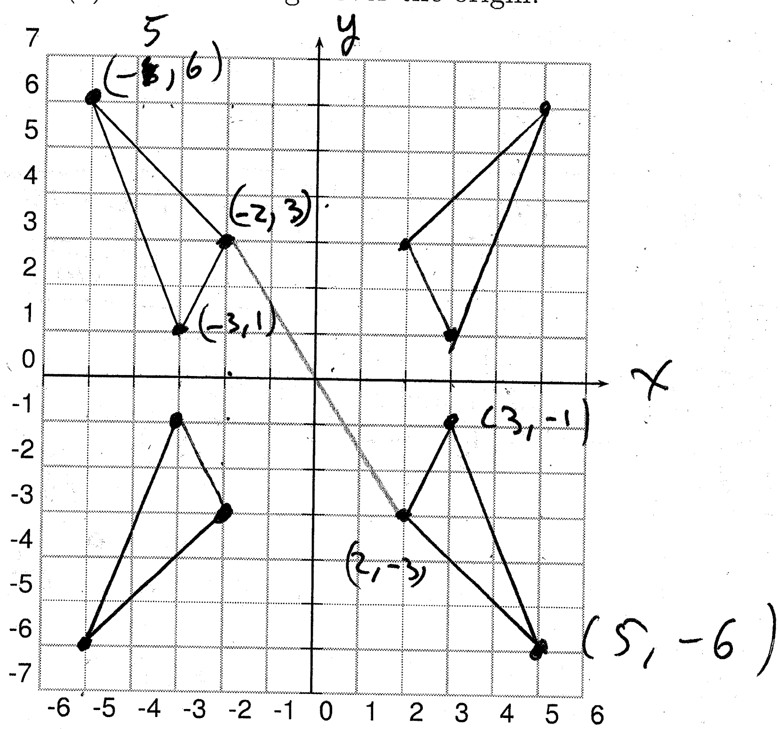
- x-axis, replace y with $-y$.
- y-axis, replace x with $-x$.
- origin, replace x with $-x$ and y with $-y$.

Example 1.1.4. Use the graph below to

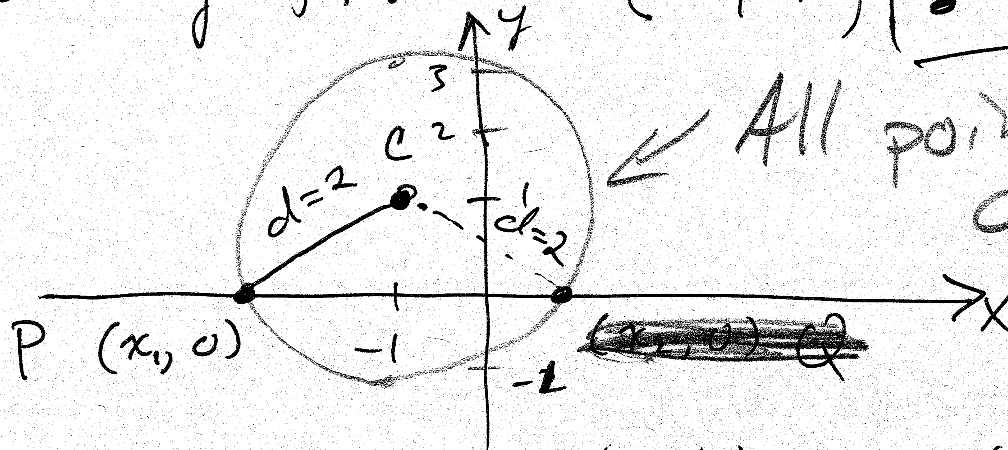
(1) Reflect the triangle over the x-axis.

(2) Reflect triangle over the y-axis.

(3) Reflect triangle over the origin.



All points on the x-axis 2 units away from $(-1, 1)$: $\boxed{(-1+\sqrt{3}, 0) \text{ \& } (-1-\sqrt{3}, 0)}$



All points 2 units away from $(-1, 1)$

distance from $\overset{(x_2, y_2)}{C(-1,1)}$ to $P\overset{(x_1, y_1)}{(x_1, 0)}$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(2)^2 = \left(\sqrt{(-1-x_1)^2 + (1-0)^2}\right)^2$$

$$-1 + 4 = (-1-x_1)^2 + 1 - 1$$

$$3 = (-1-x_1)^2$$

$$\pm\sqrt{3} = \sqrt{(-1-x_1)^2}$$

$$\pm\sqrt{3} = -1 - x_1$$

$$+1 \qquad +1$$

$$(-1)(1 \pm \sqrt{3}) = (-x_1)(-1)$$

$$\boxed{-1 \pm \sqrt{3} = x_1}$$

try numbers

$y = x^3 \leftarrow (2, 8)$

$8 = 2^3$ ✓

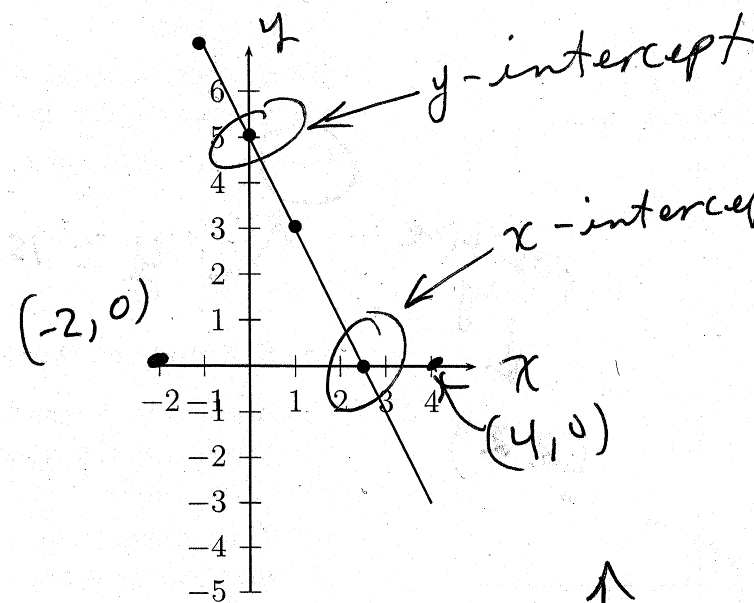
¿ $-8 = (-2)^3$ $(-2, -8)$ ¿

$-8 = -8$ ✓

1.2 Relations

We reviewed in section 1.1 how to graph points so now we want to know how to graph equations. Suppose we want to graph the equation $y = -2x + 5$. This is a relationship between x and y where the value of y is determined by they choice of x. For each x we can find a y value and that is one point (x, y) on the graph:

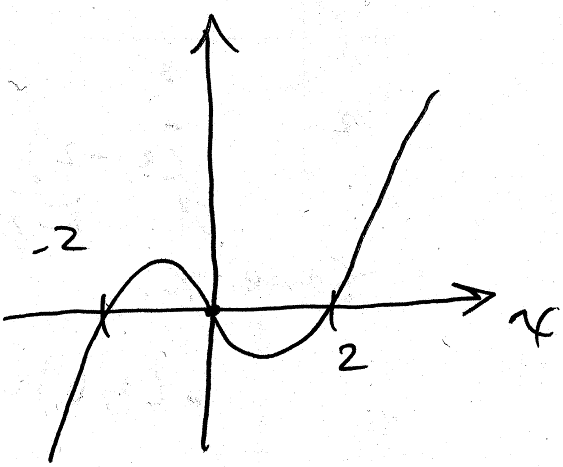
x	$y = -2x + 5$
-1	(-2)(-1)+5 = 7
0	(-2)(0)+5 = 5
1	(-2)(1)+5 = 3
2	(-2)(2)+5 = 1
5/2	(-2)(5/2)+5 = 0



x and y Intercepts

x-intercept: The point where the graph crosses the x-axis.
To find the x-intercept you set $y = 0$.

y-intercept: The point where the graph crosses the y-axis.
To find the y-intercept you set $x = 0$.



Example 1.2.1. Find all intercepts for

$$y = 4x^3 - 16x$$

y-int: set $x = 0$ $\quad y = 4(0) - 16(0) = 0$

$\boxed{(0, 0)}$

x-int: set $y = 0$ $\quad 0 = 4x^3 - 16x$ FACTOR

$0 = 4x(x^2 - 4)$

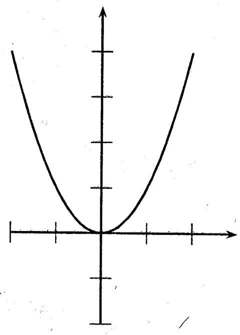
$\boxed{4x = 0}$ $\quad x^2 - 4 = 0$

$\boxed{x = 0}$ $\quad x^2 = 4$

$x = \pm\sqrt{4} = \boxed{\pm 2}$

$\boxed{\begin{array}{l}(0, 0)\\(2, 0)\\(-2, 0)\end{array}}$

Symmetry



Symmetric about y-axis

A graph is symmetric about the y-axis if it is the same on both sides of the y-axis.

Thus when (a, b) is on the graph then $(-a, b)$ is also on the graph.

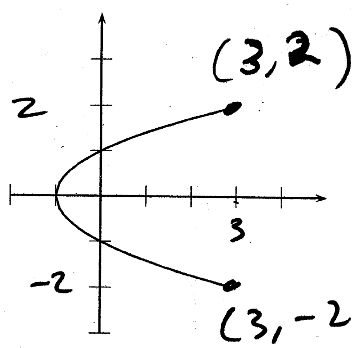
$f(x) = f(-x)$ for all x.

$f(7)$

$y = 15x^2 + 3x + 22 = f(x)$

put $x=7$ into this equation.

positive y



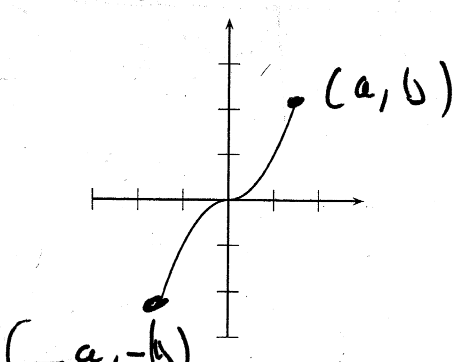
negative y

Same x.

Symmetric about x-axis

A graph is symmetric about the x-axis if it is the same on both sides of the x-axis.

Thus when (a, b) is on the graph then $(a, -b)$ is also on the graph.



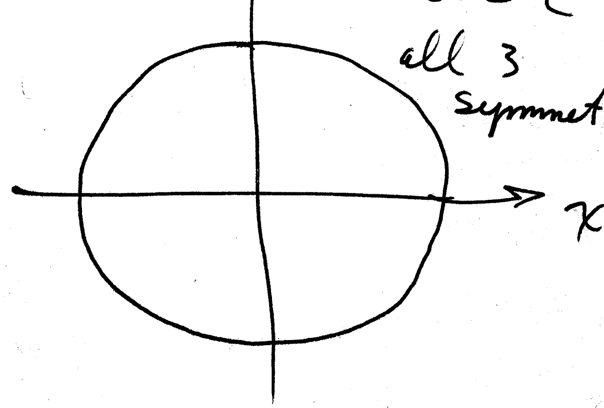
Symmetric about the origin

A graph is symmetric about the origin if the graph is unchanged by a 180 degree rotation about the origin.

Thus when (a, b) is on the graph then $(-a, -b)$ is also on the graph.

The short version

Symmetry	The equation is equivalent when ...
y-axis	x is replaced with $-x$
x-axis	y is replace with $-y$
origin	x and y are replaced by $-x$ and $-y$.



circle all 3 symmetries.

Example 1.2.2. Find the symmetry of $y = x^3$.

Try replace x with $-x$ ← y-axis symmetry?

$y = x^3$ $\qquad y = (-x)^3$

$y = -x^3$ ← Not same as original equation

x-axis symmetry

Try replace y with $-y$

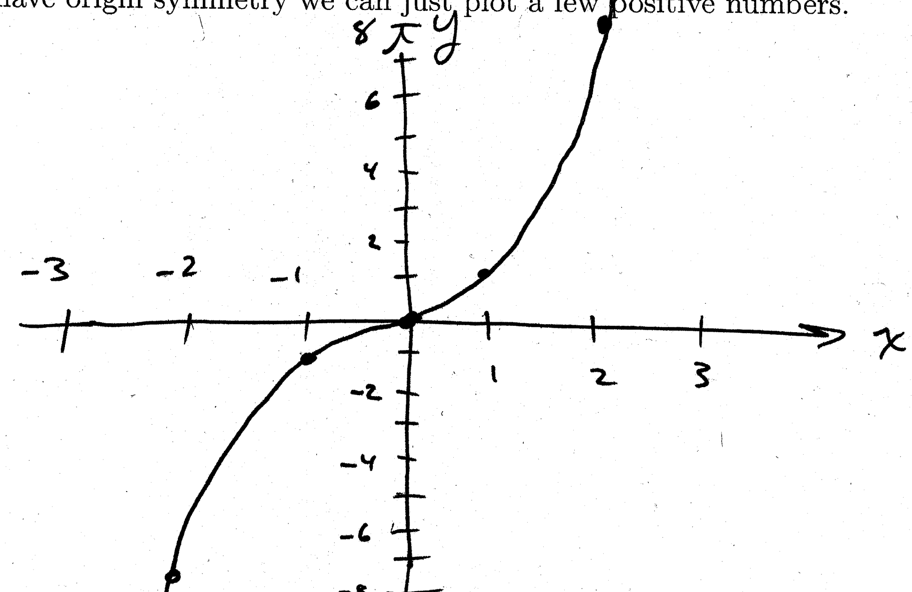
$y = x^3$ $\qquad -y = (x)^3 \Rightarrow -y = x^3$

Try replace x with $-x$ and y with $-y$

$y = x^3$ $\qquad -y = (-x)^3 \Rightarrow (-1)(-y) = (-x^3)(-1) \Rightarrow y = x^3$ ✓

Draw a sketch: Since we have origin symmetry we can just plot a few positive numbers.

x	$y = x^3$
0	0
1	1
2	8
3	27
4	64



Example 1.2.3. Find the intercepts, determine if there is any symmetry and graph the function:

$$x^2 + y^3 = 1$$

Intercepts

x-int: $y = 0$ $\qquad x^2 = 1$

$x = \pm 1$ $\qquad \boxed{(1, 0)\ (-1, 0)}$

y-int: $x = 0$ $\qquad y^3 = 1$

$y = 1$ $\qquad \boxed{(0, 1)}$

Example 1.2.4. Find the intercepts, determine if there is any symmetry and graph the function:

$$x = 2y^3 + 3y$$

$x^2+y^3=1$

symmetry

y-axis symmetry ? replace x w/ $-x$

$(-x)^2+y^3=1$

same

$x^2+y^3=1$ yes!

x-axis symmetry ? replace y w/ $-y$

$x^2+(-y)^3=1$

$x^2-y^3=1$ Not same

origin symmetry ? replace both

$(-x)^2+(-y)^3=1$

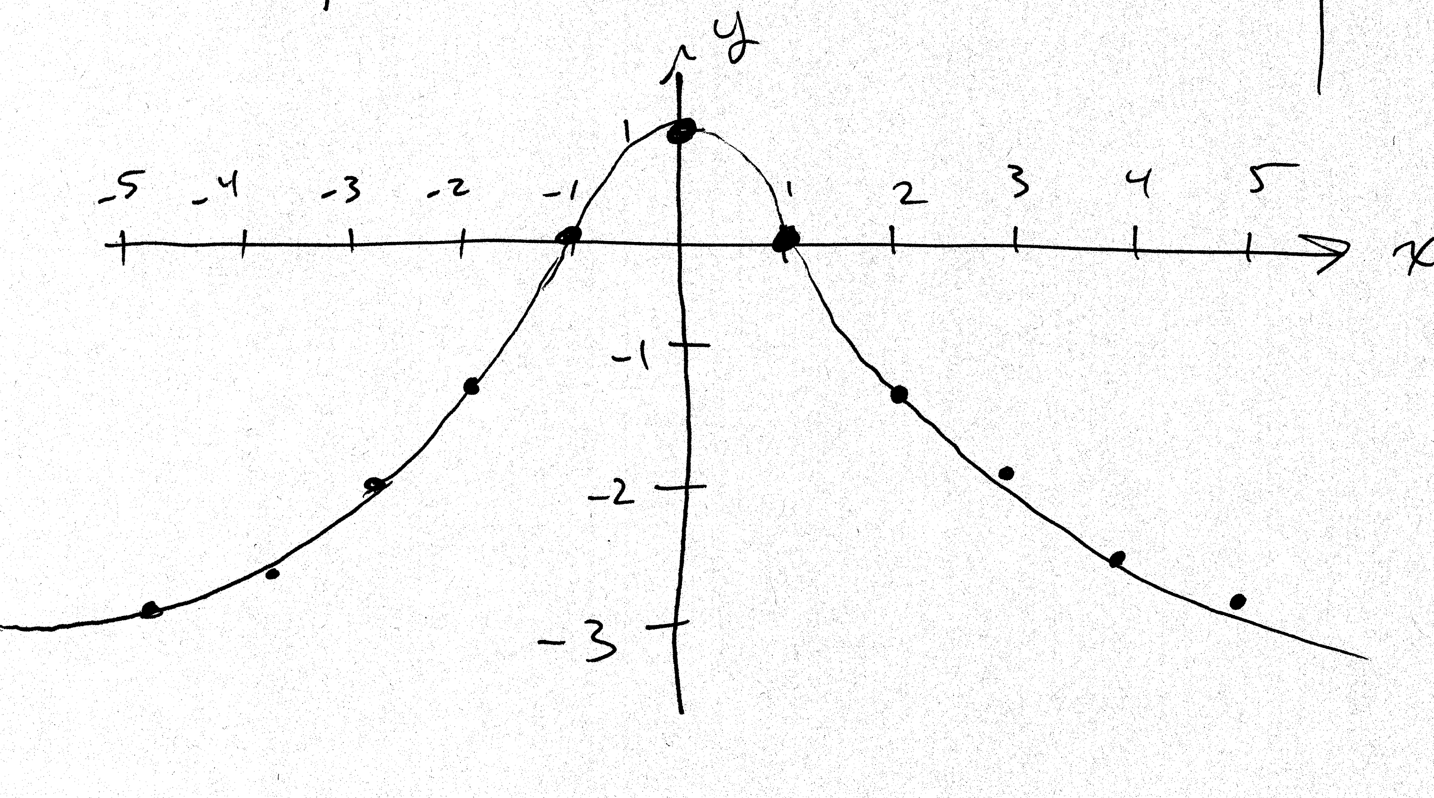
$x^2-y^3=1$ Not same

$x^2 + y^3 = 1$ ← solve for y to make life ~~easy~~ not so hard.

$$\begin{array}{r} x^2 + y^3 = 1 \\ -x^2 \qquad\quad -x^2 \\ \hline y^3 = 1 - x^2 \end{array}$$

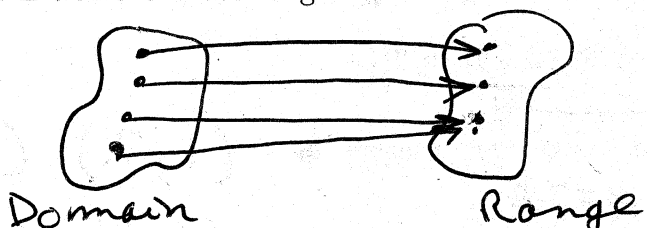
$$y = \sqrt[3]{1-x^2}$$

x	$y = \sqrt[3]{1-x^2}$	(x, y)
0	$\sqrt[3]{1-0} = 1$	$(0, 1)$
1	$\sqrt[3]{1-1} = 0$	$(1, 0)$
2	$\sqrt[3]{1-4} = \sqrt[3]{-3} = -\sqrt[3]{3} \approx -1.44$	$(2, -1.44)$
3	$\sqrt[3]{1-9} = \sqrt[3]{-8} = -2$	$(3, -2)$
4	$\sqrt[3]{1-16} = -\sqrt[3]{15} \approx -2.5$	$(4, -2.5)$
5	$\sqrt[3]{1-25} = -\sqrt[3]{24} \approx -2.9$	$(5, -2.9)$



1.3 Introduction to Functions

Definition 1.3. A **function** is a rule that establishes a correspondence between two sets of elements (called the **domain** and **range**) so that for every element in the domain there corresponds EXACTLY ONE element in the range.



Definition 1.4. A **function in one variable** is a set of ordered pairs with the property that no two ordered pairs have the same first element.

For example: { (-2, 1), (-1, 2), (0, 3), (1, 4), (2, 5) }.

Definition 1.5. Domain: The "things" you can put into a function.

Range: The "things" you get out of a function.

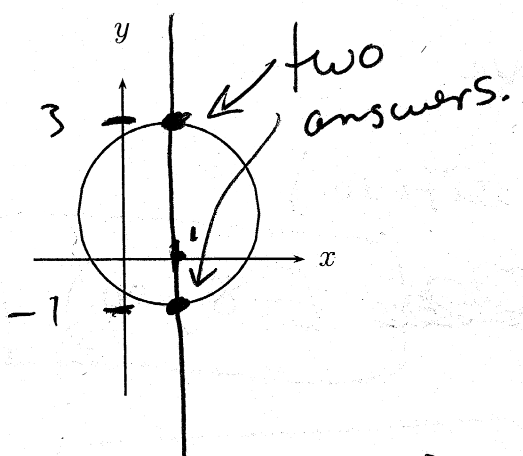
Domain $\{-2, -1, 0, 1, 2\}$ 1^{st} coordinate

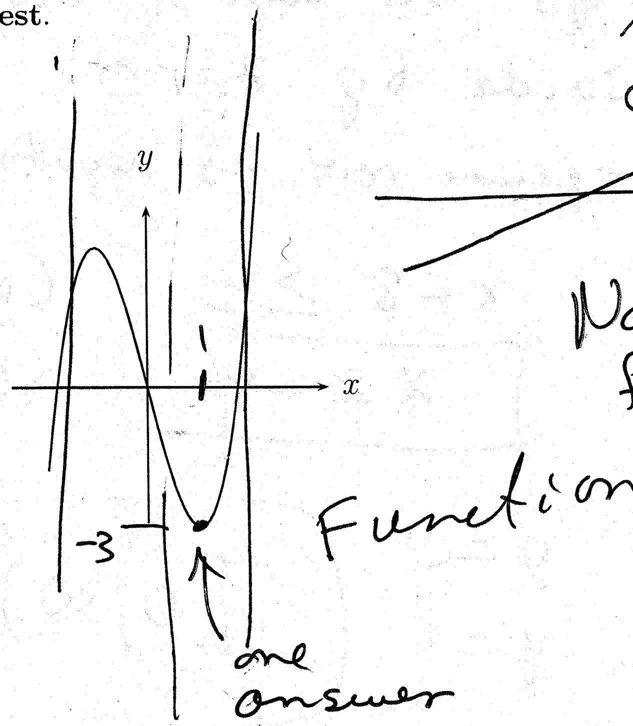
Range $\{1, 2, 3, 4, 5\}$ 2^{nd} coordinate

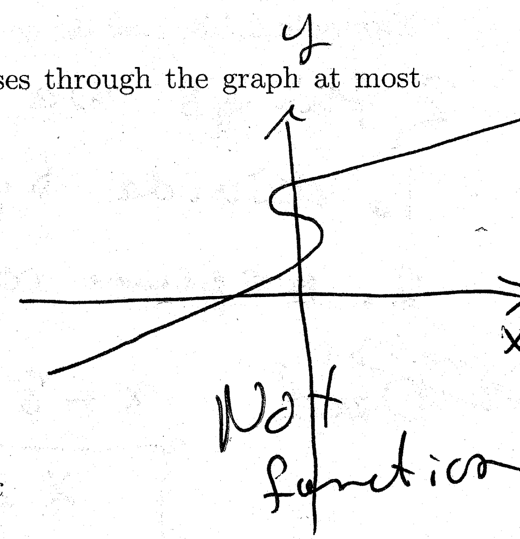
Graphically

An equation defines a function if each vertical line drawn passes through the graph at most once. This is called the **Vertical Line Test**.

For example:







$x = 1$ what is y?

$y = 3$ or $y = -1$

Not function.

Example 1.3.1. Determine whether or not the relation represents y as a function of x. Find the domain and range of those relations which are functions. Can't have repeated x-values.

F 1. $\{(-3,9),\ (-2,4),\ (-1,1),\ (0,0),\ (1,1),\ (2,4),\ (3,9)\}$

NF 2. $\{(-3,0),(1,6),(2,-3),(4,2),(-5,6),(4,-9),(6,2)\}$

NF 3. $\{(x,y)\,|\,x$ is an odd integer, and y is an even integer$\}$ $(1,2)$ $(1,4)$ $(1,6)$ $(1,8)$

$y=2x$

NF 4. $\{(-2,y)\,|\,-3<y<4\}$ $(-2,-1)$ $(-2,1)$

Example 1.3.2. Which of the following are functions of x and why?

1. $x^2+y=1$
2. $x+y^2=1$
3. $x+y^3=1$

$y=$ all x stuff in one way.

#1 $y=1-x^2$ function ✓

#2 $y^2=1-x$

$y=\pm\sqrt{1-x}$

two equations.

#3 $y^3=1-x$

$y=\sqrt[3]{1-x}$

Function.

Example 1.3.3. Find the domain and range of the function $y=\sqrt{x+8}$.

2 things we can't do.

1. divide by zero
2. square root of negative.

Domain

Need $x+8\geq 0$ (positive)

$\boxed{x\geq -8}$ $\boxed{[-8,\infty)}$

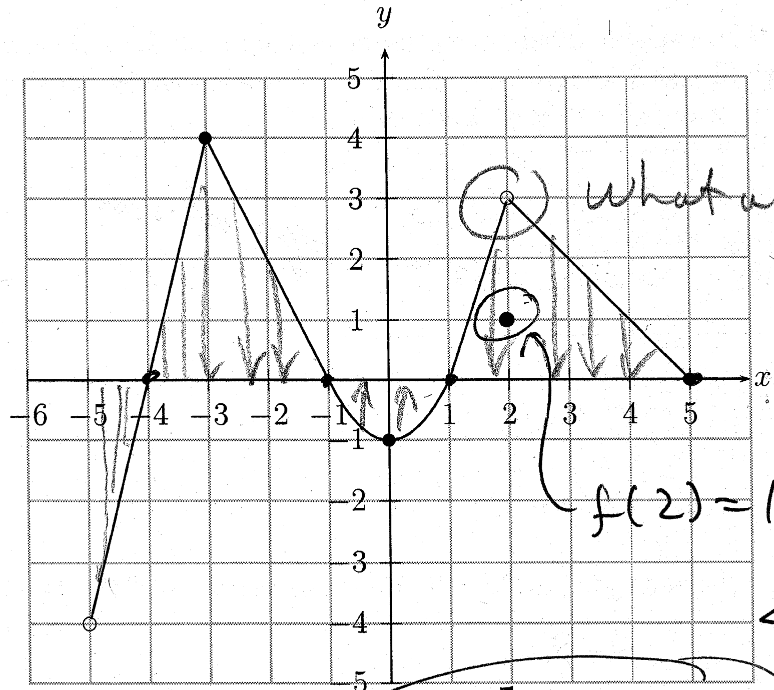
Range

$x=-8$ $y=0$

$x=-7$ $y=1$

$\boxed{[0,\infty) \text{ or } y\geq 0}$

Example 1.3.4. Use the graph of $f(x)$ below to answer the questions about $f(x)$.



← 4 is in range.

what about $y = 3$?

It is in the range.

$f(2) = 1$

← −4 not

1. Find the domain of f. $(-5, 5]$ x- values
2. Find the range of f. y - values $\boxed{(-4, 4]}$
3. Determine $f(2)$. what is the y-value when $x = 2$?
   $\boxed{f(2)=1}$
4. List the x-intercept(s), if any exist. $\boxed{x = -4, -1, 1, 5}$
5. List the y-intercept(s), if any exist. $\boxed{y = -1}$

1.4 Function Notation

We can write a function several ways. The variable used to represent elements of the Domain is the **independent variable** and the variable used to represent elements of the Range is called the **dependent variable**. The most common way of writing a function is

$$\underbrace{y}_{\text{dependent variable}} = f(\underbrace{x}_{\text{independent variable}}) = \underbrace{2x+1}_{\text{some}}.$$

equation.

We can also write a function as

$$f : x \to 2x + 1$$

or

$$f : \{(x, y) \mid y = 2x + 1\}$$

Example 1.4.1. Consider the following function: $f(x) = 7x + 3$. Find the values of $f(0)$, $f(-1)$, $f(-1+h)$ and $f(x+h)$.

$$f(0) = 7(0) + 3 = \boxed{3}$$

$$f(-1) = 7(-1) + 3 = \boxed{-4}$$

$$f(-1+h) = 7(-1+h) + 3$$

$$= \boxed{-7 + 7h + 3} = \boxed{7h - 4}$$

$$f(x+h) = 7(x+h) + 3$$

$$\boxed{= 7x + 7h + 3}$$

Example 1.4.2. Consider the following function: $f(x) = 4x^2 + 3x - 22$. Find the values of $f(0)$, $f(-1)$, and $f(x+h)$.

TEST

$f(0) = -22$ $\quad f(-1) = 4 - 3 - 22 = \boxed{-21}$

$f(x+h) = 4x^2 + 8xh + 4h^2 + 3x + 3h - 22$

$= 4(x+h)^2 + 3(x+h) - 22$

$= 4(x^2 + 2xh + h^2) + 3x + 3h - 22$

$= 4x^2 + 8xh + 4h^2 + 3x + 3h - 22$

Piecewise Functions

Example 1.4.3. Evaluate $f(0)$, $f(-1)$, $f(1)$, and $f(2)$ for

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 1 \\ 2x^2 + 2 & \text{if } x \geq 1 \end{cases}$$

$0 < 1$

$f(0) = 0^2 + 2 = \boxed{2}$ $\quad f(1) = 2(1) + 2 = \boxed{4}$

$f(-1) = 1 + 2 = \boxed{3}$ $\quad f(2) = 2(4) + 2 = \boxed{10}$

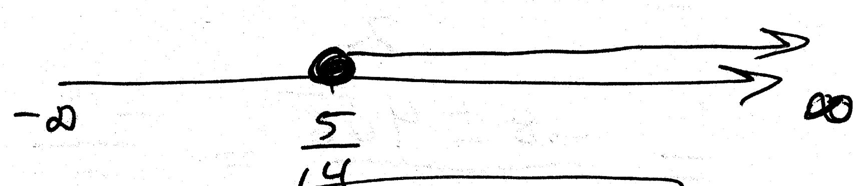
Example 1.4.4. Determine the domain for the function. Write your answer in Interval Notation and as an Inequality.

$$f(x) = -1 + \sqrt{14x - 5}$$

$14x - 5 \geq 0$

$\frac{14x}{14} \geq \frac{5}{14}$

$x \geq \frac{5}{14}$



$\boxed{\left[\frac{5}{14}, \infty\right)}$

$f(x) = 15\sqrt{14x - 5}$

Example 1.4.5. Determine the domain for the function. Write your answer in Interval Notation and as an Inequality.

$$f(x) = \frac{3x+20}{x^2+8x-8}$$

Can't divide by zero.

$$x^2+8x-8=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$ quadratic formula.

$a=1 \quad b=8 \quad c=-8$

$$x = \frac{-8 \pm \sqrt{64-4(1)(-8)}}{2(1)}$$

$$\begin{array}{r} 64 \\ 32 \\ \hline 96 \end{array}$$

$96 = 6 \cdot 16$

$$= \frac{-8 \pm \sqrt{96}}{2}$$

$$= \frac{-8 \pm \sqrt{16 \cdot 6}}{2}$$

$$= \frac{-8 \pm 4\sqrt{6}}{2} = \frac{-8}{2} \pm \frac{4\sqrt{6}}{2}$$

$$\boxed{x \neq -4 \pm 2\sqrt{6}}$$ ← Domain

$$\boxed{(-\infty, -4-2\sqrt{6}) \cup (-4-2\sqrt{6}, -4+2\sqrt{6}) \cup (-4+2\sqrt{6}, \infty)}$$

1.5 Function Arithmetic

Arithmetic Combinations

We can add, subtract, multiply and divide functions much like we do with real numbers.

Notation

$g(x) = -x^3 + 3$

$-g(x) = -(-x^3 + 3)$

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example 1.5.1. If $f(x) = 2x+3$ and $g(x) = x^2+1$ find

$$(f+g)(x) = \overbrace{2x+3}^{f(x)} + \overbrace{x^2+1}^{g(x)} = \boxed{x^2+2x+4}$$

$$(f-g)(x) = (2x+3) - (x^2+1) = 2x+3-x^2-1 = \boxed{-x^2+2x+2}$$

$$(f \cdot g)(x) = (2x+3)(x^2+1) = \boxed{2x^3+3x^2+2x+3}$$

$$\left(\frac{f}{g}\right)(x) = \boxed{\frac{2x+3}{x^2+1}}$$

We can evaluate these new functions the exact same way we did before. Whatever is in the parentheses is replaced for x in the equation.

Example 1.5.2. If $f(x) = x^2+2x-3$ and $g(x) = x^3-3x^2-4x$ find

$g(-1) = -1-3+4=0$

a) $(f+g)(-1) = f(-1) + g(-1) = -4 + 0 = \boxed{-4}$

$f(2) = 5$

b) $(f \cdot g)(2) = f(2) \cdot g(2) = 5(-12) = \boxed{-60}$

$g(2) = 8-12-8$
$= -12$

c) The domain of $\left(\frac{f}{g}\right)(x) =$

$$\frac{f(x)}{g(x)} = \frac{x^2+2x-3}{x^3-3x^2-4x}$$

set denominator $= 0$
Factor

$4 = 2 \cdot 2$
$= 4 \cdot 1$

$1-4=-3$

$$0 = x^3-3x^2-4x = x(x^2-3x-4)$$
$$= x(x+\underline{1})(x-\underline{4})$$

$\rightarrow x=0 \quad x+1=0$
$x-4=0$

$\boxed{x \neq 0 \quad x \neq -1 \quad x \neq 4}$ Domain

TEST **Example 1.5.3.** Suppose $f(x) = x^2 - 2x + 1$. Find

$$\frac{f(x+h) - f(x)}{h} \quad \text{(Difference Quotient)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{(x+h)^2 - 2(x+h) + 1}^{f(x+h)} - \overbrace{(x^2 - 2x + 1)}^{f(x)}}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h \cancel{+1} - \cancel{x^2} + \cancel{2x} - \cancel{1}}{h}$$

$$= \frac{2xh + h^2 - 2h}{h} = \frac{\cancel{h}(2x + h - 2)}{\cancel{h}}$$

$$\boxed{= 2x + h - 2}$$

$$(x+h)^2 = (x+h)(x+h)$$

Example 1.5.4. A company produces very unusual CD's for which the variable cost is \$ 7 per CD and the fixed costs are \$ 30000. They will sell the CD's for \$ 52 each. Let x be the number of CD's produced.

1. Write the total cost C as a function of the number of CD's produced. $C(x)$
2. Write the total revenue R as a function of the number of CD's produced. $R(x)$
3. Write the total profit P as a function of the number of CD's produced. $P(x)$
4. Find the number of CD's which must be produced to break even. $P(x) = 0$

1. $C(x) = 30{,}000 + 7x$

2. $R(x) = 52x$

3. $P(x) = R(x) - C(x)$
$= 52x - (30{,}000 + 7x)$
$= 45x - 30{,}000$

4. $45x - 30{,}000 = 0 \qquad x = \frac{30{,}000}{45} = \boxed{667}$

1.6 Graphs of Functions

Definition 1.6. The **graph of a function** f is a collection of ordered pairs $(x, f(x))$ such that x is in the domain of $f(x)$. All points (x, y) that make the equation true.

Recall:

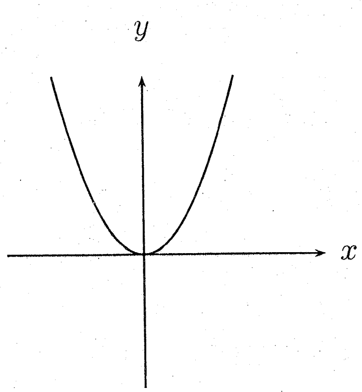
x is the distance in the x-direction. $y = f(x)$ is the the distance in the y direction.

Domain and Range

The **domain** of a function is those x-values that we can use in the function.
The **range** of a function is the y-values we get out of the function.

Example 1.6.1. $y = x^2$

Domain: All real numbers.
Range: $y \geq 0$.



$y = f(x)$

Zero's of a Function

are the x-intercepts.

set $y = 0$

crosses x-axis.

Definition 1.7. The **zero's of a function** $f(x)$ are those x-values for which $f(x) = 0$.

Q: How do we find the zero's of a function?

A: Set the function equal to zero. Also

Factor! Factor! Factor!

Example 1.6.2. Find the zero's of $f(x) = 3x^2 + 22x - 16 = 0$

$16 = 16 \cdot 1$
$= 4 \cdot 4$
$= 8 \cdot 2$

$(3x - 2)(x + 8) = 0$

$12x + 4x = 16x$ NO

$3x - 2 = 0$ or $x + 8 = 0$

$\boxed{x = \frac{2}{3} \qquad x = -8}$

Example 1.6.3. Find the zero's of $f(x) = \dfrac{x^2 - 9x + 14}{4x} = 0$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

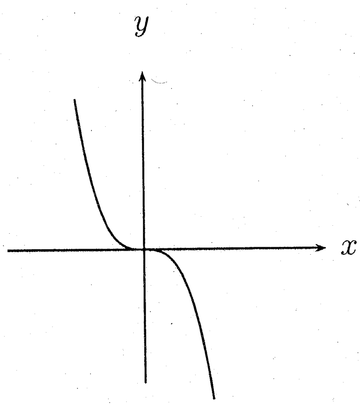
$x - 7 = 0 \qquad x - 2 = 0$

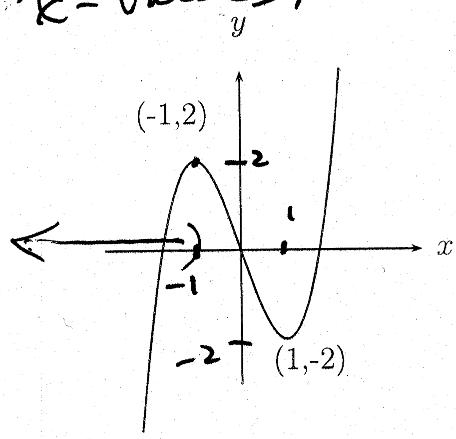
$\boxed{x = 7, \quad x = 2}$

Increasing and Decreasing Functions

Definition 1.8.
A function is **increasing** on an interval if for any x_1 and x_2 in the interval with $x_1 < x_2$ then $f(x_1) < f(x_2)$. graph goes up as we move from left to right.
A function is **decreasing** on an interval if for any x_1 and x_2 in the interval with $x_1 < x_2$ then $f(x_1) > f(x_2)$.
A function is **constant** on an interval if for any x_1 and x_2 in the interval $f(x_1) = f(x_2)$.

Example 1.6.4. Where is f(x) increasing/decreasing?
x-values.





Decreasing on
$(-\infty, \infty)$
$\mathbb{R}$ = all real numbers

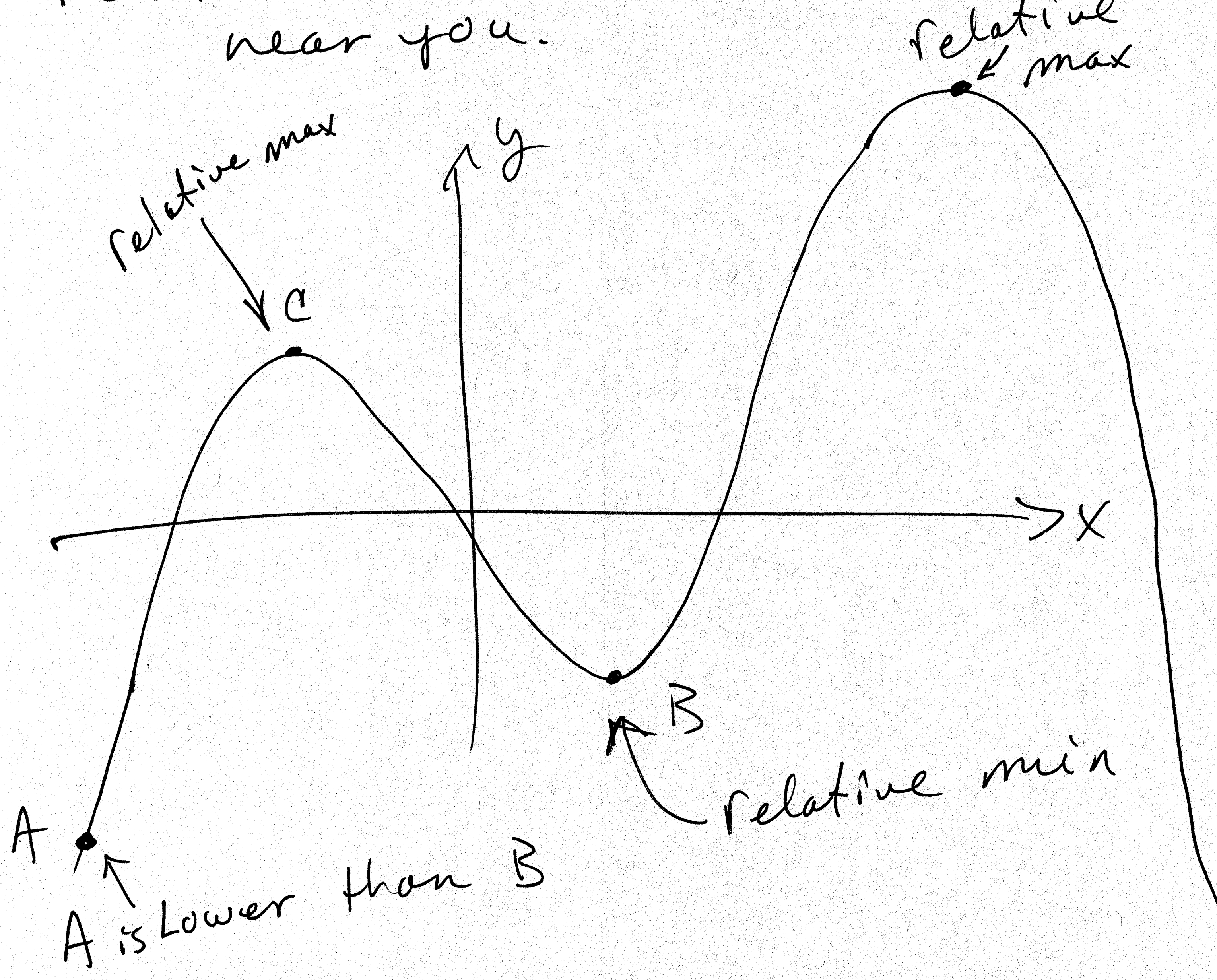
Inc $(-\infty, -1) \cup (1, \infty)$

Dec $(-1, 1)$

Maximums & Minimums

relative maximum: highest point near you.

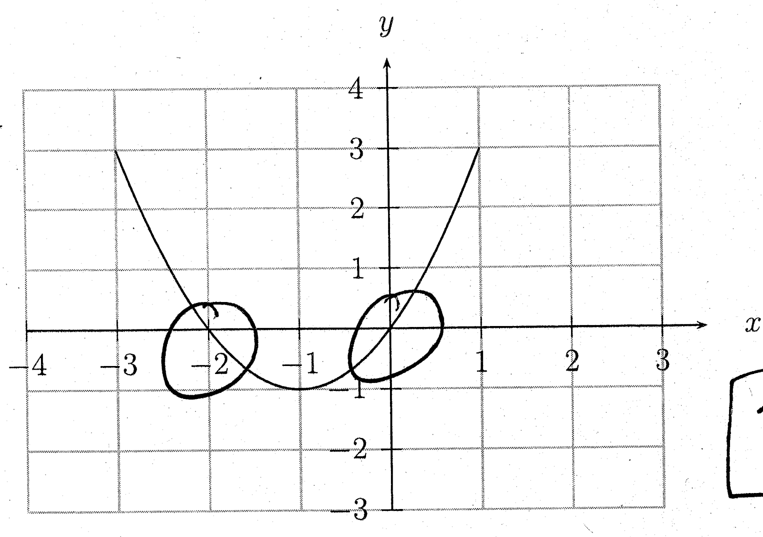
relative minimum: lowest point near you.



Example 1.6.5. Use the graph to solve the equation $x^2 + 2x = 0$

find the zeros.

Also means find the x-intercepts.



$\boxed{x = 0 \ \& \ x = -2}$

Linear Functions

$f(x) = mx + b$ Linear Function

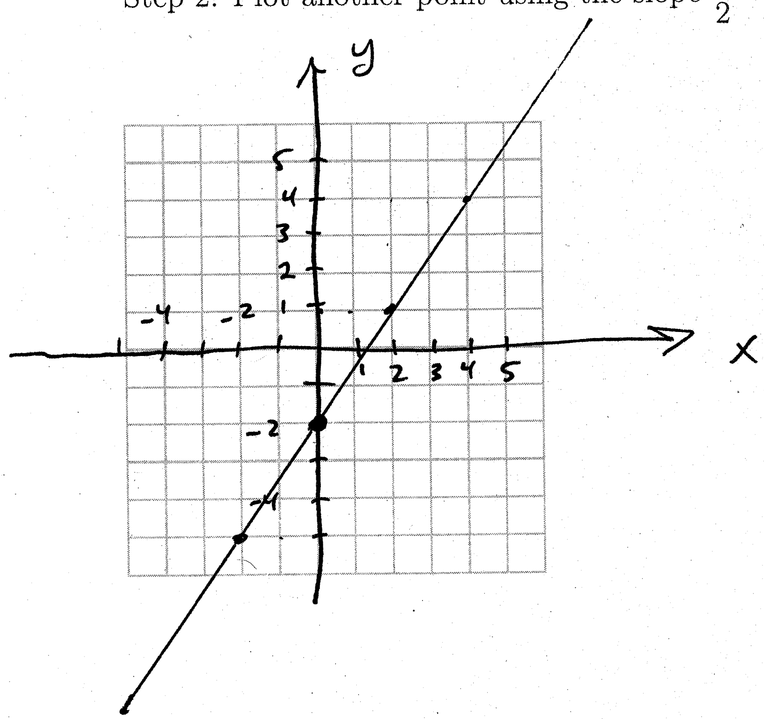
Graph: $y = mx + b$ b is y-intercept

m = slope = $= \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

Example 1.6.6. Graph $f(x) = \frac{3}{2}x - 2$

Step 1: Plot y- intercept.

Step 2: Plot another point using the slope $\frac{3}{2} = \frac{\text{rise}}{\text{run}}$



$y = \frac{3}{2}x - 2$

$2y - 3x = -4$ ←

Graphing Piecewise Functions

Example 1.6.7. Graph

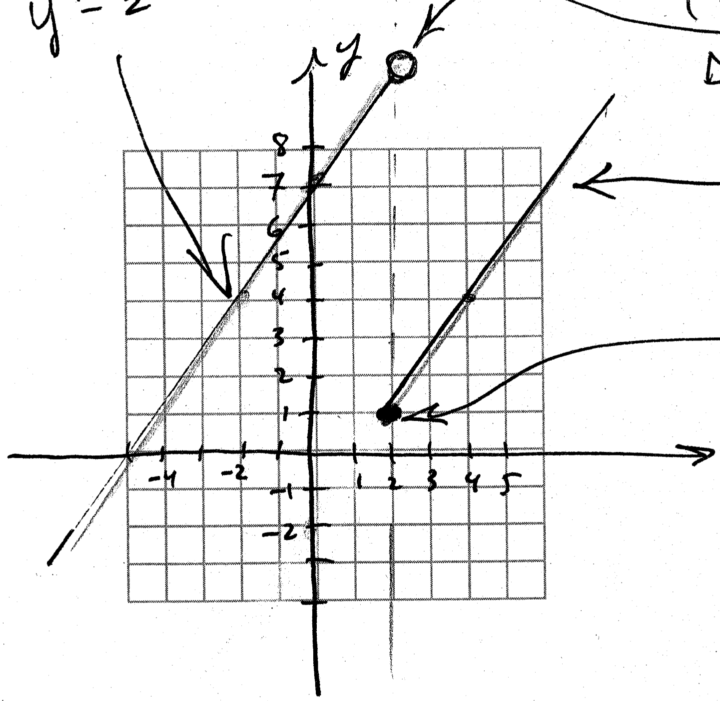
$$f(x) = \begin{cases} \frac{3}{2}x - 2 & x \geq 2 \\ \frac{3}{2}x + 7 & x < 2 \end{cases}$$

$y = \frac{3}{2}x + 7$

Don't include the endpt.

$f(2) = 1$

$y = \frac{3}{2}x - 2$



Example 1.6.8. Graph

$$f(x) = \begin{cases} -x & x \leq 0 \\ 0 & 0 < x \leq 1 \\ x - 1 & x > 1 \end{cases}$$

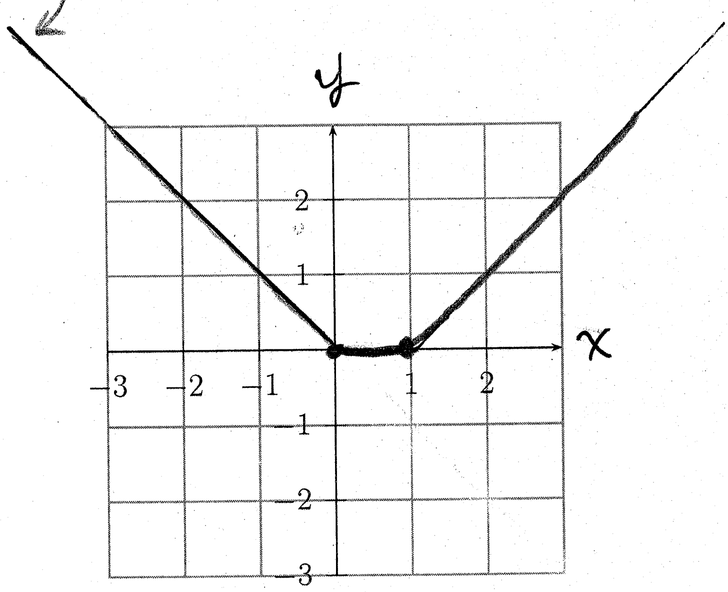
graph 3 lines

$y = -x + 0$

$y = 0$

$y = x - 1$

$y = -x$



y = a number is a horizontal line.

Even and odd functions

$f(x) = x^3$

Definition 1.9. Even is y-axis symmetry

$f(2) = 8$

A function is **even** if $f(x) = f(-x)$ for all x in the domain of $f(x)$.
A function is**odd** if $f(x) = -f(-x)$ for all x in the domain of $f(x)$.

odd is origin symmetry

$f(-2) = -8$

Example 1.6.9. Is $h(x) = x^5 - 5x^3$ even, odd or neither?

Look at $h(-x)$:

$$h(-x) = (-x)^5 - 5(-x)^3$$
$$= -x^5 + 5x^3$$
$$= -(x^5 - 5x^3) = -h(x)$$

odd

Example 1.6.10. Is $h(x) = x^4 - 3x^2$ even, odd or neither?

Look at $h(-x)$:

$$h(-x) = (-x)^4 - 3(-x)^2$$
$$= x^4 - 3x^2 = h(x)$$

even

Example 1.6.11. Is $h(x) = x^3 - 5$ even, odd or neither?

Look at $h(-x)$:

$$h(x) = x^3 - 5x^0$$

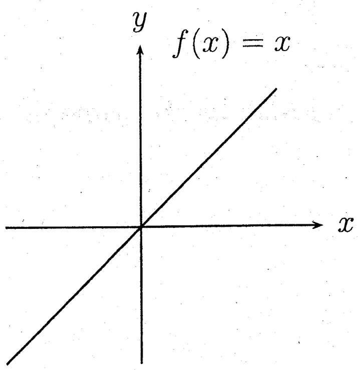
$$h(-x) = (-x)^3 - 5$$
$$= -x^3 - 5$$

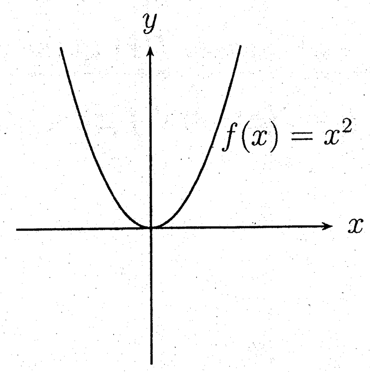
Not $-h(x)$
or $h(x)$

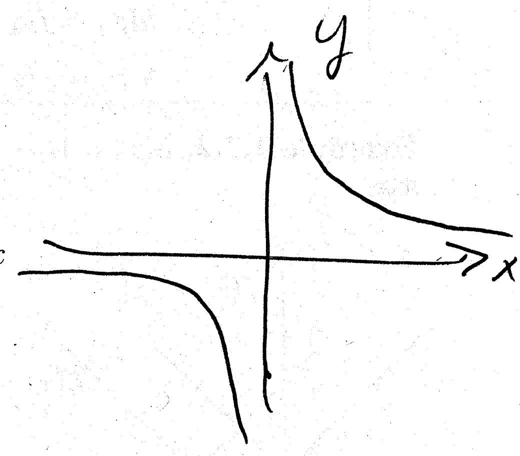
1.7 Transformations

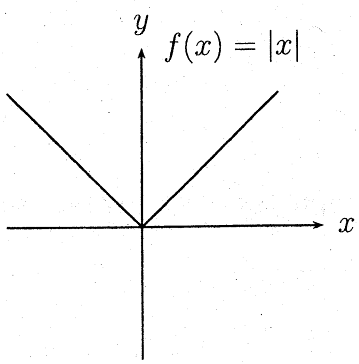
$f(x) = \frac{1}{x}$

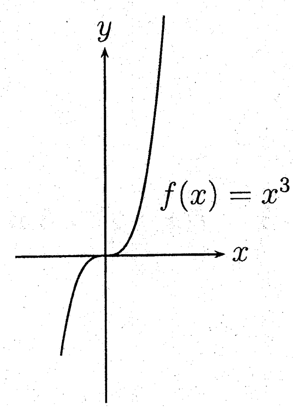
Basic Graphs



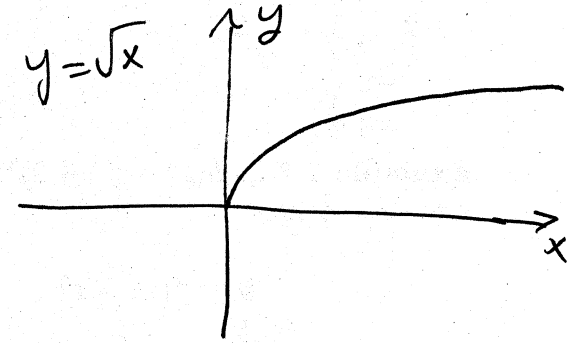








$y = \sqrt{x}$



Shifting Graphs

Moving up and down

$h(x) = f(x) + a$ moves $f(x)$ up "a" units.

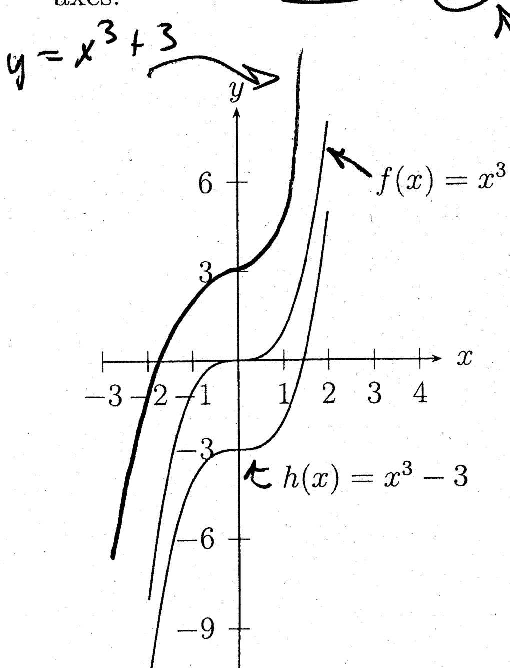
$h(x) = f(x) - a$ moves $f(x)$ down "a" units.

$y = f(x)$ Down 7

$y = f(x) - 7$

Example 1.7.1. $\boxed{h(x) = x^3 - 3} = f(x) - 3$ if $f(x) = x^3$. Graph $f(x)$ and $h(x)$ on the same set of axes.

down 3



$g(x) = x^3 + 3$

Moving left and right

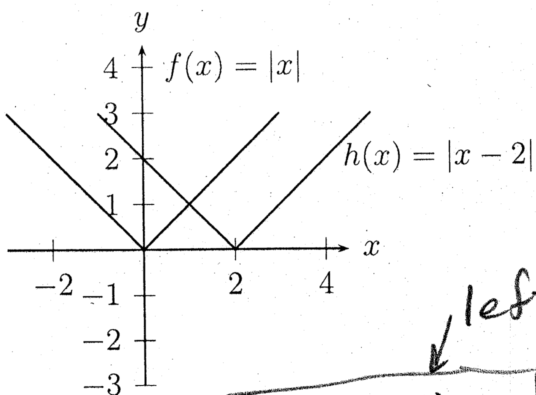
$h(x) = f(x + a)$ moves $f(x)$ to the left "a" units.

$h(x) = f(x - a)$ moves $f(x)$ to the right "a" units.

Example 1.7.2. $\boxed{h(x) = |x - 2|} = f(x - 2)$ if $f(x) = |x|$. Graph $f(x)$ and $h(x)$ on the same set of axes.

$f(x) = |x|$

$f(\underbrace{x-2}_{\text{right 2 units}}) = |x \overbrace{-2}^{\text{right 2 units}}|$



left 2

Example 1.7.3. $\boxed{h(x) = (x + 2)^3 - 3} = f(x + 2) - 3$ if $f(x) = x^3$. Graph $f(x)$ and $h(x)$ on the same set of axes.

down 3