Section 9.3 Right Angle Trigonometry:

Consider a right triangle where one of the angles is labeled $\theta$. The longest side is called the Hypotenuse ($\text{hyp}$), the side opposite the angle $\theta$ is called the Opposite Side ($\text{opp}$) and the side adjacent to the angle is called the Adjacent Side ($\text{adj}$). Using the lengths of these sides you can form 6 ratios which are the fundamental trigonometric functions of the angle $\theta$.

<table>
<thead>
<tr>
<th>Sine of $\theta$: $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$</th>
<th>Cosecant of $\theta$: $\csc(\theta) = \frac{\text{hyp}}{\text{opp}}$</th>
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<tbody>
<tr>
<td>Cosine of $\theta$: $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$</td>
<td>Secant of $\theta$: $\sec(\theta) = \frac{\text{hyp}}{\text{adj}}$</td>
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<tr>
<td>Tangent of $\theta$: $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$</td>
<td>Cotangent of $\theta$: $\cot(\theta) = \frac{\text{adj}}{\text{opp}}$</td>
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Ex 1:

\[
\begin{align*}
\sin(\theta) &= \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \\
\csc(\theta) &= \frac{\text{hyp}}{\text{opp}} = \frac{13}{5} \\
\cos(\theta) &= \frac{\text{adj}}{\text{hyp}} = \frac{12}{13} \\
\sec(\theta) &= \frac{\text{hyp}}{\text{adj}} = \frac{13}{12} \\
\tan(\theta) &= \frac{\text{opp}}{\text{adj}} = \frac{5}{12} \\
\cot(\theta) &= \frac{\text{adj}}{\text{opp}} = \frac{12}{5}
\end{align*}
\]
The same thing can be done for $\alpha$ but now the opposite side is different:

$$\sin(\alpha) = \frac{\text{opp}}{\text{hyp}} \quad \csc(\alpha) = \frac{\text{hyp}}{\text{opp}}$$

$$\cos(\alpha) = \frac{\text{adj}}{\text{hyp}} \quad \sec(\alpha) = \frac{\text{hyp}}{\text{adj}}$$

$$\tan(\alpha) = \frac{\text{opp}}{\text{adj}} \quad \cot(\alpha) = \frac{\text{adj}}{\text{opp}}$$

<table>
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<tr>
<th>Trigonometric Identities: These are always true. You must have these memorized for the test.</th>
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<td>$\sin(\theta) = \frac{1}{\csc \theta}$</td>
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<tr>
<td>$\cos(\theta) = \frac{1}{\sec \theta}$</td>
</tr>
<tr>
<td>$\tan(\theta) = \frac{1}{\cot \theta}$</td>
</tr>
<tr>
<td>$\tan(\theta) = \frac{\sin \theta}{\cos \theta}$</td>
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<th>Pythagorean identities:</th>
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<tr>
<td>$\sin^2(\theta) + \cos^2(\theta) = 1$</td>
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</table>
Ex 2: Suppose \( \tan \theta = 5 \) and \( 0 \leq \theta \leq \pi/2 \), solve for the other five trigonometric functions.

You know that \( \tan(\theta) \) is the ratio \( \frac{\text{opp}}{\text{adj}} \) so in our triangle we know that the side opposite \( \theta \) is 5 and the side adjacent is 1. We can draw a triangle and solve for the hypotenuse. Then we read the values of the trig functions from the triangle.

\[
\begin{align*}
\sin \theta &= \frac{5}{\sqrt{26}} \\
\csc \theta &= \frac{\sqrt{26}}{5} \\
\cos \theta &= \frac{1}{\sqrt{26}} \\
\sec \theta &= \frac{\sqrt{26}}{5} \\
\tan \theta &= \frac{5}{1} \\
\cot \theta &= \frac{5}{1}
\end{align*}
\]

Two Special Triangles:

For the angles 45°, 30° and 60° we have two special triangles which allow us to find their trigonometric functions. Memorize these for the test.

How do we use these two triangles?

Ex 3: Suppose we have one side of a right triangle and an angle:

We have two triangles for 60°.

\[
\sin(60) = \frac{\sqrt{3}}{2} \text{ and } \sin(60) = \frac{y}{18}
\]

So

\[
\frac{\sqrt{3}}{2} = \frac{y}{18} \Rightarrow y = 9\sqrt{3}
\]
Ex 4:

We have two triangles for $45^\circ$.

\[
sin(45) = \frac{1}{\sqrt{2}} \quad \text{and} \quad sin(45) = \frac{20}{r}
\]

So

\[
\frac{1}{\sqrt{2}} = \frac{20}{r} \Rightarrow r = 20\sqrt{2}
\]

Proving Identities

If we want to prove an identity we want to show that it is true for all values. If we have an equation and we want to know if it is an identity we work with one side and try to make it look like the other.

Ex 5. Prove the following:

a) $\cos(x)\sec(x) = 1$

We will work with the left side. Convert everything to $\cos(x)$.

\[
\cos(x)\sec(x) = \cos(x)\frac{1}{\cos(x)} = 1
\]

So the identity is true.

b) $\sin^2(x) - \cos^2(x) = 2\sin^2(x) - 1$

For this problem we will work with the left hand side again but now we need to use one of our Pythagorean identities:

\[
\sin^2(x) + \cos^2(x) = 1 \quad \Rightarrow \quad \cos^2(x) = 1 - \sin^2(x)
\]

Now we take this expression for $\cos^2(x)$ and substitute into the original equation:

\[
\sin^2(x) - \cos^2(x) = \sin^2(x) - (1 - \sin^2(x))
\]
\[
= \sin^2(x) - 1 + \sin^2(x)
\]
\[
= 2\sin^2(x) - 1
\]

So the statement is true.
Ex 6. A 6 foot person standing 20 feet from the base of a street light casts a 10 foot shadow. What is the height of the street light?