Section 5.6: Graphing

Review:

The first derivative test for increasing/decreasing.

Suppose that $f$ is continuous on $[a, b]$ and differentiable on the open interval $(a, b)$.
If $f'(x) > 0$ for all $x$ in $(a, b)$ then $f$ increases on $[a, b]$.
If $f'(x) < 0$ for all $x$ in $(a, b)$ then $f$ decreases on $[a, b]$.

The First Derivative Test for Local Extrema.

Let $f$ be a continuous function on $[a, b]$ and $c$ be a critical number in $[a, b]$.
1. If $f'(x) \geq 0$ on $(a, c)$ and $f'(x) \leq 0$ on $(c, b)$, then $f$ has a local maximum of $f(c)$ at $x = c$.
2. If $f'(x) \leq 0$ on $(a, c)$ and $f'(x) \geq 0$ on $(c, b)$, then $f$ has a local minimum of $f(c)$ at $x = c$.
3. If $f''$ does not change signs at $x = c$, then $f$ has no local extrema at $x = c$.

The Second Derivative Test for Concavity

Let $f$ be a twice differentiable function on an interval $I$.
1. If $f''(x) > 0$ on $I$, the graph of $f$ over $I$ is concave up.
2. If $f''(x) < 0$ on $I$, the graph of $f$ over $I$ is concave down.

Graphing using $y'$ and $y''$:

Steps:
1. Determine the points of discontinuity.
2. Determine the asymptotes (vertical, horizontal)
3. Determine the x- and y-intercepts.
4. Determine the critical point(s). (Set $f'(x) = 0$ and undefined).
5. Determine the intervals where the function $f$ is increasing/decreasing.
6. Determine the local extrema
7. Determine the possible point(s) of inflection. Set $f''(x) = 0$ and undefined).
8. Determine the intervals where the function $f$ is concave up/down.
9. Determine the inflection point(s)
10. Determine extra point(s) if necessary.
11. Sketch the graph using the information obtained above.
Graph the following using the steps above

**Ex 1:** $f(x) = \frac{1}{3} (x - 1)^3 + 2$
2. \( f(x) = 3x^4 + 4x^3 \)
3. \( g(x) = \frac{x}{x^2 - 4} = \frac{x}{(x - 2)(x + 2)} \)