MTH 151 - Quantifiers

Part I - Rewrite each statement using another quantifier without changing the meaning of the original statement.

1. There is at least one math student who loves logic.

2. No student doesn't love math.

3. Some days are not sunny.

4. There exists a rational number that is a real number.

5. All students are not math majors.

6. Not all integers are even.

Part II - Write the negation of each statement.

1. Some people do not pay taxes.

2. All counting numbers are divisible by 1.

3. Some rectangles are squares.

4. No even integers are divisible by five.

5. At least one integer is negative.

6. Every counting number is a positive integer.
**Euler Diagrams and Arguments With Quantifiers**

**Purposed:** The purpose of this activity is to develop the ability to use Euler diagrams to test the validity of an argument when one or more statement contains a quantifier.

**Requisites:** This activity assumes that students have been introduced to the use of Euler diagrams to represent statements with universal and/or existential quantifiers.

**Definition:** In order for an argument to be valid, the conclusion must always be true whenever the premises are true. An argument that is not valid is invalid.

**Directions:** Determine whether the following arguments (syllogisms) are valid by constructing an Euler diagram in which the premises hold. If the diagram shows the conclusion, without ambiguity, then the argument is valid. Otherwise, the argument is invalid.

Example:  
Some students are lazy.  
All males are lazy.  
Some students are males.

Consider the following Euler diagram:

![Euler Diagram Example 1]

Notice that both premises hold, but the conclusion is not shown in the diagram. Notice that the ovals for “students” and “lazy people” overlap which matches the first premise. Notice that the oval for “males” is entirely within the oval for “lazy people” which matches the second premise.

Now consider another possible Euler diagram:

![Euler Diagram Example 2]

In the above diagram, the premises hold and the conclusion is shown. However, in order for an argument to be valid, the conclusion must always be true whenever the premises are true. The premises say nothing about the relation between “students” and “males” so both diagrams must be used. To determine the validity of an argument we must consider all cases. Since the first diagram gives a case where the conclusion is not shown, even though the premises hold, the argument is invalid.

Euler diagrams are drawn from the premises only. The conclusion is tested in all cases presented by the Euler diagrams.
Construct a Euler diagram for each of the following arguments and determine the validity.

1. All students are lazy.
   *Nobody who is wealthy is a student.*
   Lazy people are not wealthy.

2. No college professor is wealthy.
   *Some poets are wealthy.*
   Some poets are not college professors.

3. All poets are interesting people.
   *Jennifer is an interesting person.*
   Jennifer is not a poet.

4. No student is lazy.
   John is an artist.
   *All artists are lazy.*
   John is not a student.

5. All expensive things are desirable.
   All desirable things make you feel good.
   *All things that make you feel good make you live longer.*
   Expensive things make you live longer.

6. Determine the validity of the following argument for each proposed conclusion.

   *All people who drive contribute to air pollution.*
   *All people who contribute to air pollution make life a little worse.*
   *Some people who live in a suburb make life a little worse.*

   a. Some people who live in a suburb drive.
   b. Some people who live in a suburb do not contribute to air pollution.
   c. Some people who contribute to air pollution live in a suburb.
   d. All people who drive make life a little worse.
   e. Some people who make life a little worse live in a suburb.
WRITING THE NEGATION OF COMPOUND STATEMENTS USING DEMORGAN'S LAWS

PURPOSE: The purpose of this activity is to develop the ability to use DeMorgan's Laws to write the negation of 2 compound statements.

REQUISITES: This activity assumes that students are familiar with the use of DeMorgan's Laws and can write compound statements in symbolic form.

DEMORGAN'S LAWS:
\[
\neg(p \lor q) \equiv \neg p \land \neg q \\
\neg(p \land q) \equiv \neg p \lor \neg q
\]

DIRECTIONS: Given a compound statement, apply DeMorgan's Laws to write the negation using the following steps:

1. Identify the simple statements within the compound statement and assign appropriate variables. Then write the compound statement in symbolic form.

2. Write the negation of the symbolic form in step #1 and use DeMorgan's Laws to write the symbolic form that is equivalent to the negation.

3. Use the symbolic form that is equivalent to the negation from step #3 to write the negation of the original compound statement in words.

Example: Given the compound statement:
John has blue eyes and Mary does not have red hair.

1. \(p: \text{John has blue eyes,} \)
\(q: \text{Mary has red hair.} \)
\(p \land \neg q\)

2. \(\neg(p \land \neg q) \equiv \neg p \lor q\)

3. John does not have blue eyes or Mary has red hair.

Complete steps 1, 2, and 3 for each of the following compound statements:

a. Your car has a flat tire and it needs a tune up.
b. \(9 - 6 = 3 \) or \(5 + 8 \neq 12.\)
c. I will not go and she will not come.
d. You will not fail the class, but he will.
e. He is not a lawyer and she is a doctor.
f. The sky is blue or the grass is red.
WRITING CONDITIONAL STATEMENTS IN SYMBOLIC FORM

PURPOSE: The purpose of this activity is to develop the ability to recognize the antecedent and the consequent in conditional statements and to then write the conditional statement in the correct symbolic form and finally in "if...then" form.

REQUISITES: This activity assumes that students know what an antecedent and consequent are and have been introduced to the various forms in which conditional statements can be written.

DIRECTIONS: Given a conditional statement:

1. Identify the antecedent with the symbol p. (Use ~ p if the antecedent is a negation.)
2. Identify the consequent with the symbol q. (Use ~ q if the consequent is a negation.)
3. Write the given conditional statement in symbolic form and then rewrite the statement in "if...then" form.

Example: Given the conditional statement:
Being a resident of Virginia is necessary for being a resident of Radford.

1. p: You are a resident of Radford.
2. q: You are a resident of Virginia.
3. p → q: If you are a resident of Radford, then you are a resident of Virginia.

Complete steps 1, 2, and 3 for each of the following conditional statements:

a. Today is Friday only if it rains.
b. You will pass the class, if you do your homework.
c. All policemen carry guns.
d. Rain is sufficient for your lawn growing.
e. A penny saved is a penny earned.
f. Going to the concert implies having enough money.
g. If you get a 95 on the test, you get an A.
h. Passing math is necessary for getting a degree.
i. You can do it, if he can do it.
j. No horses have five legs.
WRITING THE NEGATION OF A CONDITIONAL STATEMENT

_Purpose:_ The purpose of this activity is to develop the ability to write the negation of a conditional statement.

_Requisites:_ This activity assumes that students can correctly identify the antecedent and consequent in a conditional statement and write the statement in “if…then” form.

_Definition:_ The negation of a conditional statement, \( p \rightarrow q \), is \( p \land \sim q \).

_Alternate Definition:_ \( \sim (p \rightarrow q) \equiv p \land \sim q \)

_Directions:_ Given a conditional statement:

1. Identify the antecedent and the consequent.
2. Write the conditional statement in symbolic form and use the definitions above to write the negation in symbolic form.
3. Write the negation of the conditional statement in words.

Example: Given the condition statement: Tricycles are not bicycles.

1. \( p \): It is a tricycle.
   \( \sim q \): It is not a bicycle.
2. \( \sim (p \rightarrow q) \equiv p \land \sim q \)
3. Negation: It is a tricycle and it is a bicycle.

Complete steps 1, 2, and 3 for each of the following conditional statements:

a. All entomologists love bugs.
b. Today is Friday only if tomorrow is Saturday.
c. Dogs have fleas, if cats have tails.
d. If you don’t have the money, you can’t buy it.
e. Rain requires clouds.
f. Passing tests is necessary for passing a class.
g. He will stay, if you do not go.
h. Going to the movies requires having enough money.
i. No squirrels have feathers.
j. Having a license is sufficient for driving a car.
**USING TRUTH TABLES TO DETERMINE EQUIVALENT STATEMENTS**

**PURPOSE:** The purpose of this activity is to develop the ability to determine whether two statements are equivalent by use of truth tables.

**REQUISITES:** This activity assumes that students can construct truth tables.

**DEFINITION:** Two statements are equivalent if they have the same truth value in every possible situation.

**DIRECTIONS:** Given two statements, construct a truth table for each and determine whether or not the columns that were the last to be completed are exactly the same.

Example: Given the statements:
If it snows on Saturday, then I will go skiing. It does not snow on Saturday or I go skiing.

p: It snows on Saturday.
q: I go skiing.

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<th>p</th>
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<th>p → q</th>
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Statement 1 & Statement 2 are the same.
The statements are equivalent.
Determine whether or not the following statements are equivalent using truth tables.

a.  #1.  If it is not a bird, then it is a plane.
     #2.  It is not a bird and it is a plane.

b.  #1.  You can go to the concert, if you have enough money.
     #2.  You do not have enough money or you can go to the concert.

c.  #1.  Money is power or death is final.
     #2.  It is false that, money is not power and death is not final.

d.  #1.  I can, but he can not.
     #2.  If I can not, then he can.

e.  #1.  Running fast is necessary for winning the race.
     #2.  You do not win the race or you run fast.

f.  #1.  It is false that, you are old and he is young.
     #2.  If you are old, then he is not young.

g.  #1.  The grass needs cutting and the trees need trimming.
     #2.  The grass does not need cutting or the trees need trimming.
PURPOSE: The purpose of this activity is to develop the ability to write the converse, inverse, and contrapositive of a conditional statement.

REQUISITES: This activity assumes that students can correctly identify the antecedent and consequent in a conditional statement and that they are familiar with the symbolic form of the converse, inverse, and contrapositive.

DEFINITION: The converse of a $p \rightarrow q$-statement is $q \rightarrow p$.
The inverse of a $p \rightarrow q$ statement is $\neg p \rightarrow \neg q$.
The contrapositive of a $p \rightarrow q$ statement is $\neg q \rightarrow \neg p$.

DIRECTIONS: Given a conditional statement:

1. Identify the antecedent and consequent.

2. Write the conditional statement in symbolic form and use the definition above to write the converse, inverse, and contrapositive.

3. Using the symbolic form from step 2, write in words the converse, inverse, and contrapositive.

Example: Given the conditional statement: A penny saved is a penny earned.

1. $p$: A penny is saved, $q$: A penny is earned.
   (If a penny is saved, then a penny is earned.)

2. Conditional: $p \rightarrow q$.
   Converse: $q \rightarrow p$ / inverse: $\neg p \rightarrow \neg q$ / contrapositive: $\neg q \rightarrow \neg p$.

3. Converse: If a penny is earned, then a penny is saved.
   Inverse: If a penny is not saved, then a penny is not earned.
   Contrapositive: If a penny is not earned, then a penny is not saved.

Complete steps 1, 2, and 3 for each of the following conditional statements:

a. Blue Jays like peanuts.
b. Dogs have fleas only if cats have whiskers.
c. If the temperature is above $50^\circ F$, then it is not cold.
d. Good balance is necessary for skiing.
e. No Mondays are days of the weekend.
f. If the sky is not clear, there are clouds.
g. Flowers will die, if they are not watered.
h. A figure being a square implies the figure is a rectangle.
i. Not turning on the lights is sufficient for it being dark.
j. All trees have leaves.
USING TRUTH TABLES TO DETERMINE VALID ARGUMENTS

PURPOSE: The purpose of this activity is to develop the ability to write an argument symbolically and to construct a truth table to determine whether or not it is valid.

REQUISITES: This activity assumes that students can construct truth tables.

DEFINITION: An argument (syllogism) can be written as a conditional statement in which the antecedent is the conjunction of each of the premises of the argument and the consequent is the conclusion of the argument.

ALTERNATE DEFINITION: If \( p_1, p_2, p_3, \ldots \) are the premises of the argument and \( C \) is the conclusion, then the argument is the conditional statement:

\[
[p_1 \land p_2 \land p_3 \land \ldots] \rightarrow C
\]

Furthermore, the argument is valid if the conditional statement is a tautology. Otherwise the argument is invalid.

DIRECTIONS: Use the following steps to determine whether a given argument (syllogism) is valid or invalid:

1. Write each simple statement and use a variable to symbolize it.
2. Write the argument in symbols.
3. Construct a truth table for the conditional statement that represents the argument.
4. Determine whether the argument is valid or invalid.

Example: Given the argument:

If the clothes are dirty, I will wash them. I do not wash the clothes. Therefore, the clothes are not dirty.

1. \( p: \) The clothes are dirty.
   \( q: \) I will wash them.

2. \( p \rightarrow q \]
   \( \neg q \)
   \( \neg p \)
   \[ [(p \rightarrow q) \land (\neg q)] \rightarrow (\neg p) \]

3.

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<tr>
<th>( p )</th>
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4. Valid argument (Column 5 is a tautology).
Use steps 1-4 to determine whether the following arguments (syllogisms) are valid or invalid.

a. If I go to the store, then I will be late. I do not go to the store. Therefore, I will not be late.

b. If Greg Maddux pitches, then the Braves will win. The Braves lost. Therefore, Greg Maddux did not pitch.

c. Lightning will flash only if there is thunder. There is not thunder and it rains. Therefore, if there is no lightning, it will not rain.

d. Buying a new suit is sufficient for buying a new shirt. If he buys a new shirt, then he will buy a new tie. He buys a new suit. Therefore, he will buy a new tie.

e. It is Friday or it will rain. If it does not rain, she will go to the party. It is not Friday. Therefore, it will rain or she will not go to the party.

f. Dogs bark and cats meow. If dogs do not bark, birds do not sing. Birds sing. Therefore, cats meow.
MTH 151 - Chapter 3 Outline

The Chapter 3 Test on pages 149 and 150 is excellent.

Part I- Sentences

Be able to identify simple statements and translate form words to symbols
Be able to write a statement in words from the symbolic form

1. Conditional
   - converse
   - inverse
   - contrapositive
   - negation
   - disjunction - equivalent form
     Various forms to the If...then... form

2. Use DeMorgan's Laws to negate
   - a) conjunction
   - b) disjunction

3. Quantifiers
   - No (as a conditional)
   - All (as a conditional)
   - Some are Some are not
   - Other terms (ex. Everyone for all)
   - Negation

Part II - Definitions for Connectives

- negation
- conjunction
- disjunction (inclusive or)
- exclusive or
- conditional
- biconditional

Part III - Truth Values

Statement - from knowledge of simple statements (including sets of numbers) - when specific values are given (ex. p is true, q is false, etc.)
Truth tables
   equivalent statements
   compounds (2 or 3 simple statements)

Part IV - Valid/Invalid Arguments"

Euler diagrams Truth
   tables

Note:  Approximately 90% of the test questions will come from information learned
   from the 8 handouts on this chapter:

1. Writing the negation of compound statements using DeMorgan's Laws.
2. Writing conditional statements in symbolic form.
3. Using truth tables to determine equivalent statements.
4. Writing the negation of a conditional statement.
5. Writing the converse, inverse, and contrapositive of a conditional statement.
6. Euler Diagrams and arguments with quantifiers.
7. Using truth tables to determine valid arguments.
8. Quantifiers