**Math 279** Name: _____
**Spring 2008 Chalmeta**
**Test 2**

**Directions** No credit will be given for unsupported answers. Write everything that you wish to be graded on separate paper numbering each problem as you work it. Staple this sheet to the front of your papers when you are finished. Make sure your answers are CLEARLY marked. Don't be afraid to use complete English sentences. **Always** use correct mathematical notation and terminology. You may use calculators on any portion of this test; however, you may not use any symbolic or graphing abilities that your particular calculator may have, i.e. Integration is a symbolic function.

## Multiple Choice

1. (4 pts) Let $y(t)$ be the solution of

$$y'' + y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

Then as $t \to \infty$, (ans a)

(a) $y(t) \to \infty$
(b) $y(t) \to 0$
(c) $y(t) \to 1$
(d) The limit of $y(t)$ can not be determined from the information given.

2. (4 pts) Noting that $y_1(t) = e^t$ is a solution of

$$ty'' + 2(1-t)y' + (t-2)y = 0,$$

if you try to find a second solution $y_2$ in the form $y_2 = e^t v(t)$, which of the following equations must be satisfied by $v$? (ans c)

(a) $v'' + tv' + v = 0$
(b) $tv'' + 2(1-t)v' + (t-2)v = 0$
(c) $tv'' + 2v' = 0$
(d) $tv' + v = 0$

3. (4 pts) The general solution of $y^{(5)} + y''' = 0$ is (ans c)

(a) $c_1 + c_2 e^t + c_3 e^{-t} + c_4 t e^{-t} + c_5 t^2 e^{-t}$
(b) $c_1 + c_2 t + c_3 \cos t + c_4 \sin t$
(c) $c_1 + c_2 t + c_3 t^2 + c_4 \cos t + c_5 \sin t$
(d) $c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + c_5 t e^{-t}$

## "Short" Answer

4. (9 pts) Suppose an 8 lb weight is attached to a spring and stretches the spring 1 foot. The system has a damping coefficient of 2.5; the weight is pulled down 8 inches and released.

   (a) Write the complete initial value problem and then solve it only far enough to describe the motion of the system with the terminology of this course.
   (ans. $\frac{1}{4}y'' + 2.5y' + 8y = 0, \quad y(0) = 2/3, \quad y'(0) = 0, \quad$ under damped)

   (b) Find the coefficient of resistance for which the motion would be critically damped.
   (ans. $\gamma = \sqrt{8}$)

5. (9 pts) Find the largest interval on which the following initial value problem is guaranteed to have a unique solution:

$$(t^2 - 9)y'' + 2t(t-4)y' + 3ty = t^{-2}e^{-t}, \quad y(2) = -1, \; y'(2) = 3$$

   (ans. $0 < t < 3$)

6. (9 pts) Use variation of parameters to find the general solution to the differential equation $y'' + \dfrac{y'}{t} - \dfrac{y}{t^2} = 1$ given that $y_1(t) = t$ and $y_2(t) = t^{-1}$ are both solutions to the homogeneous equation $y'' + \dfrac{y'}{t} - \dfrac{y}{t^2} = 0$.
   (ans. $y = C_1 t + C_2 t^{-1} + \frac{1}{3}t^2$)

7. (9 pts) Find an appropriate form for $y_p$ if the method of undetermined coefficients were used to solve each of the following **(DO NOT SOLVE.)**

   (a) $y'' + 3y' + 2y = t^2e^{-t} + e^{-2t}$ **(DO NOT SOLVE.)**

   (ans. $y_p = t(A_1t^2 + A_2t + A_3)e^{-t} + Bte^{-2t}$)

   (b) $y'' + 4y = 3t\sin 2t + 4e^{2t}$ **(DO NOT SOLVE.)**

   (ans. $y_p = t(A_1t + A_2)\cos(2t) + t(A_3t + A_4)\sin(2t) + Be^{2t}$)