Math 279 Name: _____
Spring 2008 Chalmeta
Test 1

Directions No credit will be given for unsupported answers. Write everything that you wish to be graded on separate paper numbering each problem as you work it. Staple this sheet to the front of your papers when you are finished. Make sure your answers are CLEARLY marked. Don't be afraid to use complete English sentences. **Always** use correct mathematical notation and terminology. You may use calculators on any portion of this test; however, you may not use any symbolic or graphing abilities that your particular calculator may have, i.e. Integration is a symbolic function.

Multiple Choice

For the multiple choice problems you don't need to show any work. (3 pts each)

1. The general solution of $y' = y^2t^2$ has the form (ans c)

   (a) $y = \pm\sqrt{e^{t^3/3+C}}$.

   (b) $y = \dfrac{y^3t^2}{3} + C$.

   (c) $y = \dfrac{-3}{t^3 + C}$.

   (d) $y = \dfrac{C}{t}$.

2. Which of the following differential equations are linear? (ans c)

   (A) $y'' - \dfrac{1}{y} = 0$

   (B) $(1 - x^2)yy' = x$

   (C) $y'' - e^x y' + (1 + x^2)y = \tan x$

   (D) $y''' - 2y'' + 3(1 - y') = x^2 + y$

   (a) Only (C)

   (b) Only (A) and (C)

   (c) Only (C) and (D)

   (d) None of the differential equations

3. If $y(t)$ satisfies $ty' + y = t$ with $y(1) = 1$, then y(2) equals (ans a)

   (a) $\frac{5}{4}$

   (b) 2

   (c) e^2

   (d) $\frac{1}{e}$

Written Response

4. Mixing problem

   (a) (5 pts) A tank with a capacity of 500 gallons originally contains 200 gallons of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. **SET UP** (but **do not solve**) the differential equation that would allow one to determine the amount of salt in the tank at any time. Carefully define any variables you are using (with units) and state all appropriate initial conditions in terms of those variables. (ans Q = lbs of salt, $Q(0) = 100g$ $Q' = 3 - \frac{2Q}{200+t}$)

   (b) (2 pts) Suppose $y = f(t)$ is the solution to the problem in part (a). How would you determine the concentration of salt in the tank at the moment when the tank is full? (ans $f(300)$)

5. (8 pts) An object having a mass of 0.15 kg is thrown vertically upward from a 30 m tower with an initial velocity of 20 m/sec. Air resistance acting on the object is $|v|/30$, where the velocity v is measured in m/sec. When does the object reach its maximum height? (ans $t = 1.68sec$)

6. (7 pts) Consider the initial value problem $y' = 2ty + 3$, with $y(2) = 4$. Use Euler's method with step size 1 to find a very crude approximation for $y(4)$.

7. Given the initial value problem $xy' + 2y = x^2 - x + 1, \quad y(1) = \frac{1}{4}$.

   (a) (4 pts) Find the largest interval on which the initial value problem will be guaranteed to have a unique solution. (ans $(0, \infty)$)

   (b) (4 pts) Solve the initial value problem. (ans $y = \frac{1}{4}x^2 - \frac{1}{3}x + \frac{1}{2} - \frac{1}{6x^2}$)