Math 279 Name: _____
Spring 2007 Chalmeta
Test 1

Directions No credit will be given for unsupported answers. Write everything that you wish to be graded on separate paper numbering each problem as you work it. Staple this sheet to the front of your papers when you are finished. Make sure your answers are CLEARLY marked. Don't be afraid to use complete English sentences. **Always** use correct mathematical notation and terminology. You may use calculators on any portion of this test; however, you may not use any symbolic or graphing abilities that your particular calculator may have, i.e. Integration is a symbolic function.

Multiple Choice

1. What is the largest open interval contianing $t = 2$ in which the solution of the initial value problem $(t^2 - 9)y' + (t - 1)y = 3\ln(|\ t\ |)$, is certain to exist? (ans c)

   (a) $(0, \infty)$ (b) (-3, 3) (c) (0, 3) (d) (1, 3)

2. Let $y(t)$ be the solution of the intitial value problem $y' - 2y = t,\ y(0) = 1$. Then $y(1)$ equals?(ans b)

   (a) $-\frac{1}{4} + e^2$ (b) $-\frac{3}{4} + \frac{5}{4}e^2$ (c) $\frac{1}{4} + \frac{1}{4}e^2$ (d) $\frac{1}{2}$

3. Let $y(t)$ be defined as the solution of the equation $ty' + y = \cos t$ for all $t > 0$ satisfying $\lim_{t\to 0} y(t) = 1$. Then (ans c)

   (a) $y(\pi) = \frac{1}{\pi}$ (b) $y(\pi) = 1$ (c) $y\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$ (d) None of the above

Written Response

4. Find an explicit solution for $y' + x^3y^2 = y^2,\ y(0) = -1$. (ans $y = \frac{4}{x^4 - 4x - 4}$)

5. A 1,000 liter tank is initially half full of a dye solution containing 0.1 grams of dye per liter. A different dye solution containing 0.05 grams of dye per liter flows in at 5 liters per minute and the well-stirred mixture flows out at 3 liters per minute.

   (a) Set up **(but do not solve)** the differential equation that would allow one to determine the amount of dye in the tank at any time. Carefully define any variables you are using (with units) and state all appropriate initial conditions in terms of those variables. (ans Q = grams of dye, $Q(0) = 50g\ Q' = .25 - \frac{3Q}{500+2t}$)

   (b) Suppose $y = f(t)$ is the solution to the problem in part (a). How would you determine the concentration of dye in the tank at the moment when the tank is full? (ans $\frac{f(250)}{1000}$ grams/liter)

6. An object having a mass of 5 kg is thrown vertically upward from a 15 m tower with an initial velocity of 7 m/sec. Air resistance acting on the object is equal to six tenths of its velocity. When does the object reach its maximum height? (ans $t = .69sec$)

7. Find a 2-step Euler approximation for y(1.2) if $y' = 1 - x + 4y,\ y(1) = 2$.