Properties of exponents

Let a and b be positive numbers with $a \neq 1$, $b \neq 1$ and let x and y be real numbers. Then:

A) Exponent Laws:

1. $a^x a^y = a^{x+y}$
2. $(a^x)^y = a^{xy}$
3. $(ab)^x = a^x b^x$
4. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
5. $\frac{a^x}{a^y} = a^{x-y}$

Properties of Logarithms

Let b be a positive real number with $b \neq 1$, and let x be any real number. Then:

1. $\log_b(1) = 0 \qquad$ i.e. $b^0 = 1$
2. $\log_b(b) = 1 \qquad$ i.e. $b^1 = b$
3. $\log_b(b^x) = x \qquad$ i.e. $b^x = b^x$
4. $b^{\log_b(x)} = x$ if $x > 0$
5. $\log_b(MN) = \log_b(M) + \log_b(N)$
6. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$
7. $\log_b(M^p) = p\log_b(M)$
8. $\log_b(M) = \log_b(N) \iff M = N$

The natural logarithm

This is the same as before but now we use base e. Since the log base e shows up so often we call it the **natural log**.

$$\log_e(x) = \ln(x)$$

We also use log base 10 very often so we abbreviate that as

$$\log_{10}(x) = \log(x).$$

Your calculator follows the same convention.

Change of Base Formula

Let a, b, x be positive real numbers with $a \neq 1, b \neq 1$. Then

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)} \qquad \text{(For any } b\text{)}$$

For the calculator you can use either base 10 or base e.

$$\log_a(x) = \frac{\log(x)}{\log(a)} \qquad \text{OR} \qquad \log_a(x) = \frac{\ln(x)}{\ln(a)}.$$