

13 Consumer Mathematics

13.1 The Time Value of Money

Start with some **Definitions**:

Definition 1. The amount of a loan or a deposit is called the **principal**.

Definition 2. The amount a loan or a deposit increases over time is called the **interest**. The interest is usually computed as a percentage of the principal. This percentage is called the **interest rate**.

Definition 3. Interest calculated only on principal is called **simple interest**. Interest calculated on principal plus any previously earned interest is called **compound interest**.

Simple Interest

If P = principal, r = annual interest rate, and t = time (in years), then the **simple interest** I is given by

$$I = Prt.$$

Example 1. Find the simple interest owed if you borrowed \$1,400 at 8% for 1 year.

Example 2. Find the simple interest owed if you borrowed \$1,460 at 7.82% for 22 months.

Example 3. Find the simple interest paid to Tony Soprano if you borrowed \$10,000 at 50% for 6 months.

Future and Present Values for Simple Interest

If a principal P is borrowed at simple interest for t years at an annual interest rate of r , then the **future value** of the loan, denoted A , is given by

$$A = P(1 + rt).$$

Likewise, the **present value**, P , is given by

$$P = \frac{A}{1 + rt}.$$

Example 4. Chris Campbell opened a security service on March 1. To pay for office furniture and guard dogs, Campbell borrowed \$14,800 at the bank and agreed to pay the loan back in 10 months at 9% simple interest. Find the *total amount* required to repay the loan.

Example 5. What is the maximum amount you can borrow today if it must be repaid in 4 months with simple interest at 8% and you know that at that time you will be able to repay no more than \$1500.

Future and Present Values of Compound Interest

If P dollars are deposited at an annual interest rate of r , compounded m times per year, and the money is left on deposit for a total of n periods, then the **future value**, A (the final amount on deposit), is given by

$$A = P \left(1 + \frac{r}{m} \right)^n,$$

and the **present value**, P , is given by

$$P = \frac{A}{\left(1 + \frac{r}{m} \right)^n}$$

Example 6. For each of the following deposits, find the future value (final amount on deposit) and the interest when compounding occurs (a) annually, (b) semiannually, and (c) monthly

Principal	Rate	time
\$1000	10%	3 years
\$3000	5%	7 years
\$15,000	7%	9 years

The effective annual yield, sometimes called the annual percentage yield (APY), is a way to compare nominal interest rates compounded for different time intervals. APY is the amount of interest you would have earned in one year if the loan had paid simple interest rather than compound interest. The APY is usually somewhat higher than the nominal rate.

Effective Annual Yield (Annual Percentage Yield, APY)

A nominal interest rate of r compounded m times per year, is equivalent to the following **effective annual yield**.

$$Y = \left(1 + \frac{r}{m}\right)^n - 1$$

Example 7. A passbook savings account pays a nominal rate of 10% on savings deposits. Find the effective annual yield if the interest is compounded semiannually. (ans 10.3%)

Example 8. Suppose a savings and loan pays a nominal rate of 4% on savings deposits. Find the effective annual yield if interest is compounded monthly. (ans. 4.1%)

Example 9. A passbook savings account pays a nominal rate of 5% on savings deposits. Find the effective annual yield if the interest is compounded 1000 times per year. (ans 5.1%)

Future Value for Continuous Compounding

If an initial deposit of p dollars earns continuously compounded interest at an annual rate of r for a period of t years, then the **future value, A** , is calculated as follows.

$$A = Pe^{rt}$$

Both **inflation** and **deflation** are calculated continuously.

Example 10. The year 2010 price of a fast food meal at a certain restaurant is \$6.12. Find the estimated future price for 2015 and 2025 with 4% inflation and with 9% inflation. (ans. \$7.74, \$11.15, \$9.60, \$23.61)

13.2 Consumer Credit

Start with some **Definitions**:

Definition 1. Borrowing to finance purchases, and repaying with periodic payments is called **installment buying**.

Definition 2. There are two types of installment buying.

- The first type, **closed-end credit**, involves borrowing a set amount up front and paying a series of equal payments until the loan is paid off. These are sometimes called **fixed installment loans**.
- With the second type, **open-end credit**, there is no fixed number of payments. The consumer continues to pay until no balance is owed. A credit card would be an example of this type of credit.

Closed End Credit

Loans set up under closed end credit often are based on **add-on interest**. This behaves like a simple interest loan. If P = principal, r = annual interest rate, and t = time (in years) then

$$\begin{aligned}\text{Amount to be repaid} &= \text{Amount borrowed} + \text{Interest due} \\ &= P + Prt.\end{aligned}$$

Example 1. Suppose you buy appliances costing \$2150 at a store charging 12% add-on interest, and you make a \$500 down payment.

1. Find the total amount you will be financing.
2. Find the total interest if you will pay off the loan over a 2-year period.
3. Find the total amount owed.
4. The amount from # 3 is to be repaid in 24 monthly installments. Find the monthly payment.

Example 2. Suppose you want to buy a new car that costs \$16,500. You have no cash- only your old car, which is worth \$3000 as a trade-in.

1. How much do you need to finance to buy the new car?
2. The dealer says the interest rate is 9% add-on (simple interest) for 3 years. Find the total interest.
3. Find the total amount owed.
4. Find the monthly payment.

Open-End Credit

- The consumer makes purchases on a credit card and at the end of the billing period receives an **itemized billing** which details all the purchases made that month, cash advances, total balance owed, minimum payment required and perhaps other information.
- Any charges beyond cash advances and purchases are called **finance charges**. They can include interest, annual fees, credit insurance coverage, carrying charges, etc..

There are two ways to calculate finance charges:

1. **unpaid balance method** (This one is very uncommon.)
2. **average daily balance method**

Example 3. Unpaid balance method

Complete the following table showing the unpaid balance at the end of each month. Assume a 1.1% interest rate on the unpaid balance.

Month	Unpaid Balance at Beginning of Month	Finance Charge	Purchases During Month	Returns	Payment	Unpaid Balance at End of Month
February	\$319.10		\$86.14	0	\$50	
March			\$109.83	15.75	\$60	
April			\$39.74	0	\$72	
May			\$56.29	18.09	\$50	

Example 4. Average Daily Balance Method

Using the average daily balance method and the information given, find (a) the average daily balance, (b) the monthly finance charge, and (c) the account balance for the next billing.

Previous balance: \$728.36

May 9 billing date

May 17 Payment \$200

May 30 Dinner \$46.11

June 3 Theater Tickets \$64.50

Date	Running Balance	Number of days Until Balance Changed	$\left(\begin{smallmatrix}\text{Running} \\ \text{Balance}\end{smallmatrix}\right) \times \left(\begin{smallmatrix}\text{Number} \\ \text{of days}\end{smallmatrix}\right)$
May 9			
May 17			
May 30			
June 3			

$$\text{Average Daily Balance} = \frac{\text{Sum of daily balances}}{\text{Days in billing period}} =$$

Example 5. Average Daily Balance Method

Using the average daily balance method and the information given, find (a) the average daily balance, (b) the monthly finance charge, and (c) the account balance for the next billing.

Previous balance: \$983.25

August 17	billing date	
August 21	Mail order	\$14.92
August 23	Returns	\$25.41
August 27	Beverages	\$31.82
August 31	Payment	\$108
September 9	Returns	\$71.14
September 11	Concert tickets	\$110
September 14	Cash advance	\$100

Date	Running Balance	Number of days Until Balance Changed	$\left(\frac{\text{Running}}{\text{Balance}}\right) \times \left(\frac{\text{Number}}{\text{of days}}\right)$
August 17			
August 21			
August 23			
August 27			
August 31			
September 9			
September 11			
September 14			

Example 6. Beth Johnson's bank card account charges 1.1% per month on the average daily balance as well as the following special fees:

Cash advance fee: 2% (not less than \$2 nor more than \$10)

Late payment fee: \$15

Over-the-credit-limit fee: \$5

In the month of June, Beth's average daily balance was \$1943. She was on vacation during the month and did not get her account payment in on time, which resulted in a late payment and resulted in charges accumulating to a sum above her credit limit. She also used her card for five \$100 cash advances while on vacation. Find the special fees charged to the account based on the account transaction in that month. (ans. \$30.00)

Example 7. Dorothy Laymon is considering two bank card offers that are the same in all respects except for the following.

Bank A charges no annual fee and charges monthly interest of 2.48% on the unpaid balance.

Bank B charges a \$25 annual fee and monthly interest of 2.13% on the unpaid balance.

From her records, Dorothy has found that the unpaid balance she tends to carry from month to month is quite consistent and averages \$990. Estimate her total yearly cost to use the card if she chooses the from **(a)** Bank A **(b)** Bank B.

13.3 Truth in Lending

Start with some **Definitions**:

Definition 1. The true annual interest rate is called the **annual percentage rate** or **APR**. All sellers must disclose the APR if you ask and print it in the contract even if you don't ask.

See handout for discussion on Table 6: Annual Percentage Rate (APR) for Monthly Payment Loans

Example 1. Finding true annual interest rate

Find the APR (true annual interest rate), to the nearest half percent, for each of the following.

Amount Financed	Finance Charge	Number of Monthly payments
\$1000	\$75	12
\$6600	\$750	30

Purchase Price	Down Payment	Add-on Interest rate	Number of payments
\$4190	\$390	6%	12
\$7480	\$2200	5%	18

Example 2. Finding the monthly payment

Find the monthly payment for each of the following.

Purchase Price	Down Payment	Finance Charge	Number of Monthly payments
\$3000	\$500	\$250	24
\$3950	\$300	\$800	48
\$8400	\$2500	\$1300	60

13.4 Purchasing a House

Start with some **Definitions**:

Definition 1. A loan for a substantial amount, extending over a lengthy time interval, for the purpose of buying a home or other property or real estate, and for which the property is pledged as security for the loan, is called a **mortgage** or **deed of trust**.

Definition 2. The time until final payoff is called the **term**.

Definition 3. The portion of the purchase price for the home which the buyer pays initially is called the **down payment**.

Definition 4. The **principal amount of the mortgage** is the amount borrowed. You find it by subtracting the down payment from the purchase price.

Fixed-rate Mortgages

Definition 5. A **fixed-rate mortgage** has an interest rate that will remain constant throughout the term of the loan.

Regular Monthly Payments

The **regular monthly payment** required to repay a loan of P dollars, together with interest at an annual rate r , over a term of t years, is given by

$$R = \frac{P \left(\frac{r}{12} \right) \left(1 + \frac{r}{12} \right)^{12t}}{\left(1 + \frac{r}{12} \right)^{12t} - 1}$$

Example 1. Find the monthly payment needed to repay each of the following fixed-rate mortgages. Use the formula above and Table 4.

Loan Amount	Interest Rate	Term
\$70,000	10.0%	20 years
\$85,000	8.0%	30 years
\$132,500	7.5%	25 years

Definition 6. An **Amortization Schedule** (or **repayment schedule**) shows how much of the payment goes toward principal and how much goes toward interest in each month of the loan. Early in the loan more of the payment goes toward interest and at the end most of the payment goes toward principal.

Calculating a repayment schedule:

$$\textbf{Step 1:} \text{ Interest for the month} = \left(\begin{array}{c} \text{Old balance} \\ \text{of principal} \end{array} \right) \left(\begin{array}{c} \text{Annual} \\ \text{interest rate} \end{array} \right) \left(\frac{1}{12} \text{ year} \right)$$

$$\textbf{Step 2:} \text{ Payment on principal} = \left(\begin{array}{c} \text{Monthly} \\ \text{payment} \end{array} \right) - \left(\begin{array}{c} \text{Interest for} \\ \text{the month} \end{array} \right)$$

$$\textbf{Step 3:} \text{ New balance on principal} = \left(\begin{array}{c} \text{Old Balance} \\ \text{of principal} \end{array} \right) - \left(\begin{array}{c} \text{Payment on} \\ \text{principal} \end{array} \right)$$

Example 2. Complete a repayment schedule for the first 3 months of a mortgage of \$87,000 at an interest rate of 6% if the term of the loan is 30 years. Calculate the monthly payment for each month and then find the amount paid toward principal and interest each month.

Example 3. Find the total monthly payment, including taxes and insurance on a mortgage of \$72,890 with an interest rate of 5.5%. The term of the loan is 15 years, the annual taxes are \$1850 and the annual insurance is \$545.

Example 4. Refer to the following table of closing costs for the purchase of a \$175,000 house requiring a 20% down payment.

Title insurance premium	\$240
Document recording fee	30
Loan fee (two points)	_____
Appraisal fee	225
Prorated property taxes	685
Prorated fire insurance premium	295

1. find the mortgage amount
2. find the loan fee
3. find the closing costs

Adjustable-rate Mortgages

Adjustable rate mortgages start with an interest rate and then change that rate yearly after a specified period. Typically 1, 3, 5, 7, or 10 years. The frequency of the change is called the adjustment period.

index this is the base rate used to calculate the interest of the loan.

margin this is the rate added to the index to find the actual APR for the loan.

Example 5. The table shows the specifications of an adjustable rate mortgage. Assume no caps apply.

Beginning Balance	\$75,000
Term	20 years
Initial index rate	6.5%
Margin	2.5%
Adjustment period	1 year
Adjusted index rate	8.0%
Adjusted balance	\$73,595.52

1. Find the initial monthly payment.
2. Find the monthly payment for the second adjustment period.
3. Find the change in monthly payment at the first adjustment. (The "adjusted balance" is the principal balance at the time of the first rate adjustment.)