Section 5.5 The Fibonacci Sequence and the Golden Ratio:

The *Fibonacci Sequence* is the following:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610,…..

Each term in the Fibonacci sequence is the sum of the previous two. The next three terms in the sequence are…..?

We can write each term of the sequence mathematically. We say that $F_n$ represents the $n^{th}$ term in the Fibonacci sequence and we write:

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$

$F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.

Ex 1: Recall that $F_n$ represents the Fibonacci number in the $n^{th}$ position in the sequence. What are the only two values of $n$ such that $F_n = n$?

Ex 2: If two successive terms of the Fibonacci sequence are both odd, is the next term even or odd?

Ex 3: A pattern is established involving terms of the Fibonacci sequence. Use inductive reasoning to make a conjecture concerning the next equation in the pattern, and verify it.

a) $1 = 2 - 1$
   $1 + 3 = 5 - 1$
   $1 + 3 + 8 = 13 - 1$
   $1 + 3 + 8 + 21 = 34 - 1$
   $1 + 3 + 8 + 21 + 55 = 89 - 1$

\[
\begin{align*}
1^2 + 1^2 &= 2 \\
1^2 + 2^2 &= 5 \\
\end{align*}
\]

b) $2^2 + 3^2 = 13$
   $3^2 + 5^2 = 34$
   $5^2 + 8^2 = 89$
The Golden Ratio:

Consider the quotients of the first few terms of the Fibonacci numbers:

\[
\begin{align*}
\frac{1}{1} &= 1 \\
\frac{2}{1} &= 2 \\
\frac{3}{2} &= 1.5 \\
\frac{5}{3} &= 1.666... \\
\frac{8}{5} &= 1.6 \\
\frac{13}{8} &= 1.625 \\
\frac{21}{13} &\approx 1.615384615 \\
\frac{34}{21} &\approx 1.619047619 \\
\frac{55}{34} &\approx 1.617647059 \\
\frac{89}{55} &= 1.618181818
\end{align*}
\]

The quotients are approaching a number close to 1.618. In fact they approach the number

\[\phi = \frac{1 + \sqrt{5}}{2}.\]

This number \(\phi\) is called the golden ratio.

Ex 4: What is the exact value of the golden ratio?

Ex 5: What is the approximate value of the golden ratio to the nearest thousandth?