2.1 Symbols and Terminology

Definitions:

- A set is a collection of objects.
- The objects belonging to the set are called **elements**, or **members**, of the set.

Sets can be designated in one of three different ways:

1. Word description:

The set of even counting numbers less than ten

- 2. The listing method
 - $\{2, 4, 6, 8\}$
- 3. Or set builder notation.
 - $\{x \mid x \text{ is an even counting number less than } 10\}$

Sets of Numbers:

Natural or counting numbers: $\{1, 2, 3, \ldots\}$

Whole numbers: $\{0, 1, 2, 3, ...\}$

Integers: {..., -1, -2, -3, 0, 1, 2, 3, ...}

Rational numbers: $\{p/q \mid p \text{ and } q \text{ are integers, and } q \neq 0\}$:

Real numbers: $\{x \mid x \text{ is a number that can be written as a decimal}$

Irrational numbers: $\{x \mid x \text{ is a real number and } x \text{ cannot be written as a quotient of integers.} \}$

Definitions:

- The number of elements in a set is called the **cardinal number**, or **cardinality**, of the set. The symbol n(A), which is read "n of A," represents the cardinal number of the set A.
- If the cardinal number of a set is a whole number then we call that set a **finite set**.
- If the set has so many elements that it cannot be represented by a whole number we say the set is an **infinite set**.

Examples

| Example 1. | Match | each s | set in | column | I with | the | appropriate | description | in | column | II |
|------------|----------|---------|---------|--------|--------|------|-------------|-------------|-----|--------|----|
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| | A. The set of all even integers |
|--|---|
| | B. The set of the five least postive integer powers of 2 |
| 1. $\{x \mid x \text{ is an even integer greater than } 4$ and less than 6 $\}$ | C. The set of even positive integers less than 10 |
| 2. $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$ | D. The set of all odd integers |
| 3. $\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$ | E. The set of all negative integers |
| 4. $\{1, 3, 5, 7, 9\}$ | F. The set of odd positive integers less than 10 |
| | G. Ø |
| | H. The set of the five least positive integer multiples of 2 |

Example 2. List the elements of the set. Use set notation to describe the set:

- 1. The set of all whole numbers greater than 8 and less than 18
- 2. The set of all counting numbers between 4 and 14

Example 3. Denote each set by the listing method.

- 1. The set of all counting numbers greater than 20.
- 2. The set of all integers between -200 and 500
- 3. $\{x \mid x \text{ is an odd integer between -8 and 7}\}$

Example 4. Identify each set as finite or infinite.

- 1. $\{2, 4, 6, \ldots, 32\}$
- 2. $\{-10, -8, -6, \ldots\}$
- 3. $\{x \mid x \text{ is a natural number less than } 50\}$

Example 5. Find n(A) for each set.

- 1. $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$
- 2. $A = \{2, 4, 6, \dots, 1000\}$
- 3. $A = \{a, b, c, \dots, z\}$
- 4. $A = \{x \mid x \text{ is a vowel in the English alphabet}\}$

We use the symbol \in to indicate "is an element of".

Example 6. $3 \in \{1, 2, 3, 4, 5\}$

We use the symbol $\not\in$ to indicate "is not an element of".

Example 7. $7 \notin \{1, 2, 3, 4, 5\}$

We say set A is equal to set B provided the following two conditions are met:

- 1. Every element of A is an element of B, and
- 2. Every element of B is an element of A

In other words two sets are equal if they contain the same elements regardless of order.

Example 8. Write "true" or "false" for each of the following statements.

$$3 \in \{2, 5, 6, 8\}$$

$$m \in \{l, m, n, o, p\}$$

$$6 \in \{-1, -2, -3, -4, -5, 6\}$$

$$9 \notin \{6, 3, 4, 8\}$$

$$\{k, c, r, a\} = \{k, c, a, r\}$$

$$\{5, 8, 9\} = \{5, 8, 9, 0\}$$

$$\{x \mid x \text{ is a natural number less than } 3\} = \{1, 2\}$$

$$\{x \mid x \text{ is a natural number greater than } 10\} = \{11, 12, 13, \ldots\}$$

2.2 Venn Diagrams and Subsets

Definitions:

- The Universal set, U, is the set of everything you are talking about.
- Venn Diagrams are drawings used to represents sets. In a Venn diagram the universal set is represented by a rectangle and any oval inside the rectangle represents some piece of the universal set. The inside of the oval is given a name, for example A, and the outside of the oval is called A' (read "'A prime"').
- Complement of A: We say that A' is the complement of A. It contains all the elements of U that are not in A. In set notation it is written as

$$A' = \{ x \mid x \in U \text{ and } x \notin A \}.$$

• A set A is a **subset** of set B if every element of A is also an element of B. In symbols we write

$$A \subseteq B$$
.

- If A and B are sets then we say A and B are equal if $A \subseteq B$ and $B \subseteq A$. We write A = B.
- Set A is a **proper subset** of set B if $A \subseteq B$ and $A \neq B$. In symbols:

 $A \subset B$.

- The number of subsets of a set of n elements is 2^{n} .
- The number of proper subsets of a set of n elements is $2^n 1$.

Examples

Suppose

$$A = \{a, b, c, d, e\}$$

$$B = \{a, c, e, b, d\}$$

$$C = \{a, c, d\}$$

$$D = \{a, c, f\}$$

then the following are true:

$$A = B, A \subseteq B, B \subseteq A, C \subseteq B, C \subseteq A, C \subset B, D \not\subseteq A$$

Insert \subseteq , \subset , and/or $\not\subseteq$ in each blank to make the statement true.

- 1. $\{-2, 0, 2\}$ _____ $\{-1, -1, 1, 2\}$
- 2. $\{2,5\}$ {0,1,5,3,4,2}
- 3. $\{\emptyset\}$ $\{a, b, c, d, e\}$
- 4. $\{B, C, D\}$ $\{B, C, D, F\}$
- 5. $\{-1, 0, 1, 2, 3\}$ {0, 1, 2, 3, 4}

Tell whether each statements 1 - 5 are True or False. Answer the rest of the questions.

- Let $U = \{a, b, c, d, e, f, g\}$, $A = \{a, e\}$, $B = \{a, b, e, f, g\}$, $C = \{b, f, g\}$, $D = \{d, e\}$ 1. $A \subset U$ 2. $D \subseteq B$ 3. $A \subset B$ 4. $\emptyset \subset A$
- 5. $D \not\subseteq B$
- 6. Find the number of subsets of C. Find the number of proper subsets of C.
- 7. Find the number of subsets of A. Find the number of proper subsets of A.
- 8. Find the number of subsets of \emptyset .
- 9. Draw a Venn Diagram representing the relationship among the sets B, C, and U.

2.3 Set Operations and Cartesian Products

Definitions:

• The intersection of sets A and B, written $\mathbf{A} \cap \mathbf{B}$ is the set of elements common to both A and B. In symbols we write

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}.$$

- **Disjoint sets** are sets that have no elements in common. Specifically $A \cap B = \emptyset$.
- The **union** of sets *A* and *B*, written **A** ∪ **B** is the set of all elements belonging to either of the sets. In symbols we write

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}.$$

• The difference of sets A and B, written $\mathbf{A} - \mathbf{B}$, is the set of all elements belonging to set A and not to set B. In symbols it is written as

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}.$$

- When two items are **paired** the order is important. To distinguish an ordered pair from a set we use parentheses () instead of braces {}. In the ordered pair (a, b), a is called the first component and b is called the second component. In general (a, b) ≠ (b, a).
- The Cartesian Product of sets A and B, written $\mathbf{A} \times \mathbf{B}$, is

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

• The cardinal number of a cartesian product If n(A) = a and n(B) = b, then

$$n(A \times B) = n(B \times A) = n(A) \times n(B) = n(B) \times n(A) = ab$$

• De Morgan's Laws: For any sets A and B,

$$(A \cap B)' = A' \cup B'$$
 and $(A \cup B)' = A' \cap B'$

Examples

Suppose

$$U = \{a, b, c, d, e, f, g\}$$

$$X = \{a, c, e, g\}$$

$$Y = \{a, b, c\}$$

$$Z = \{b, c, d, e, f\}$$

then perform the indicated operation:

- 1. $X \cap Y$
- 2. $Y \cup Z$
- 3. $X \cup U$
- 4. X'
- 5. $X' \cap Y'$
- 6. $X \cup (Y \cap Z)$.
- 7. $(Y \cap Z') \cup X$.

Use a Venn Diagram to shade each of the following sets.





2.4 Cardinal Numbers and Surveys

The Cardinal Number Formula: For any two sets A and B

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Examples

1. Use the numerals representing the cardinalities in the Venn diagrams to give the cardinality of each set specified.



- (a) $n(A \cap B)$
- (b) $n(A \cup B)$
- (c) $n(A' \cap B)$
- (d) $n(A' \cap B')$
- 2. Use the numerals representing the cardinalities in the Venn diagrams to give the cardinality of each set specified.



- (a) $n(A \cap B \cap C)$
- (b) $n(A \cap B \cap C')$
- (c) $n(A' \cap B \cap C)$
- (d) $n(A' \cap B' \cap C)$
- (e) $n(A' \cap B' \cap C')$

- 3. Evaluate $n(A \cup B)$ if n(A) = 8, n(B) = 14, and $n(A \cap B) = 5$.
- 4. Use the given information to fill in the number of elements for each region (w, x, y, z) in the Venn diagram:



 $n(A) = 18, n(B) = 15, n(A \cap B) = 11 \text{ and } n(B') = 30$

- 5. Paula Story is a fan of the music of Paul Simon and Art Garfunkel. In her collection of 22 compact discs, she has the following:
 - 5 on which both Simon and Garfunkel sing

8 on which Simon sings

7 on which Garfunkel sings

12 on which neither Simon nor Garfunkel sings.

- (a) How many of her CDs feature only Paul Simon?
- (b) How many of her CDs fearture only Art Garfunkel?
- (c) How many of the CDs feature at least on of these two artists?



- 6. The following list shows the movies viewed by a group of 55 students.
 - 17 had seen The Natural
 - 17 had seen Field of Dreams
 - $23~\mathrm{had}$ seen The Rookie
 - 6 had seen The Natural and Feild of Dreams
 - 8 had seen The Natural and The Rookie
 - 10 had seen Field of Dreams and The Rookie
 - 2 had seen all three of these movies



How many students had seen:

- (a) Exactly two of these movies?
- (b) exactly one of these movies?
- (c) none of these movies?
- (d) The Natural but none of the others?

7. It was once said that Country-Western songs emphasize three basic themes: love, prison, and trucks. A survey of the local Country-Western radio station produced the following data:

12 songs about a truck driver who is in love while in prison

13 about a prisoner in love

28 about a person in love

18 about a truck driver in love

3 about a truck driver in prison who is not in love

2 about people in prison who are not in love and do not drive trucks

8 about people who are out of prison, are not in love, and do not drive trucks

16 about truck drivers who are not in prison



(a) How many songs were surveyed?

Find the number of songs about:

- (b) truck drivers
- (c) prisoners
- (d) truck drivers in prison
- (e) people not in prison
- (f) people not in love