Unit Conversions

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Abstract

Conversion between different units of measurement is one of the first concepts covered at the start of a course in chemistry or physics. Unfortunately, unit conversions are also one of the most confusing concepts to many students. Because unit conversions are used throughout the sciences, it is crucial to understand them from the start. Hopefully, I can help clear up some of the confusion surrounding this topic. Please email me with questions, comments, or corrections.

1 Introduction

The most important thing to understand about converting from one unit of measurement to another is that we never change the *quantity* or *dimension* involved. For example, I am 6 feet 2 inches tall. I'm also 74 inches tall; 18.5 hands tall; 187.96 centimeters tall. The point is that changing the unit of measurement does not change my height.

I classify unit conversions into two categories: direct and indirect*. Direct conversions are the simplest, and are the basis for indirect conversions, so let's examine those first.

2 Direct Conversions

A direct conversion is simply one in which you convert directly from one unit to another—no intermediate steps required. In order to perform a *direct* conversion, I need an equation which directly relates my starting and ending units. For example, if you need to convert from feet to inches, you need the equation $1\text{ft} = 12\text{in}$. This equation states that one foot is the same length as 12 inches. The equals sign, "=", means that the quantity to the left of the equals sign is the same as the quantity to the right of the equals sign. Now consider this: since 1ft and 12in are actually the same quantity, then the fraction (or ratio) $\frac{1\text{ft}}{12\text{in}}$ is equal to 1. (Anytime you divide a quantity by itself, you get 1.) Also, $\frac{12\text{in}}{1\text{ft}} = 1$. These two ratios are called *unit multipliers*. Since unit multipliers equal one, we

*You probably won't see these terms anywhere else. They are simply terms I use to help clarify the process.

can multiply a measurement by one of them and not change the quantity—after all, we are multiplying times one. The only problem now is knowing which unit multiplier to use. Fortunately, there is an easy way—just pick whichever one cancels out the unit you don't want.

Here's an example: I want to convert 6 feet to inches, and I know that 1ft=12in. So I have two possible unit multipliers:

$$\frac{1\text{ft}}{12\text{in}} \quad \text{and} \quad \frac{12\text{in}}{1\text{ft}}$$

Which do I use? Let's try the first one first:

$$\frac{6\text{ft}}{1} \times \frac{1\text{ft}}{12\text{in}} = \frac{6\text{ft} \times 1\text{ft}}{1 \times 12\text{in}} = \frac{6\text{ft}^2}{12\text{in}} = \frac{1}{2}\frac{\text{ft}^2}{\text{in}}$$

That's just basic multiplication of fractions. Wait a minute! We wanted inches, not feet squared per inch... that doesn't even make sense! So let's try the other multiplier:

$$\frac{6\text{ft}}{1} \times \frac{12\text{in}}{1\text{ft}} = \frac{6\text{ft} \times 12\text{in}}{1 \times 1\text{ft}} = \frac{6 \times 12}{1}\frac{\cancel{\text{ft}} \cdot \text{in}}{\cancel{\text{ft}}} = 72\text{in}$$

Notice how the ft cancel because they are in both the numerator and the denominator. This leaves us with just inches, which is exactly what we wanted.

As another example, let's convert my 74 inches of height into hands, given that 1 hand = 4 inches. (I'll abbreviate hand as "h".) The two unit multipliers are

$$\frac{1\text{h}}{4\text{in}} \quad \text{and} \quad \frac{4\text{in}}{1\text{h}}$$

Since we are converting from inches to hands, we want to use the unit multiplier that will cancel inches:

$$\frac{74\text{in}}{1} \times \frac{1\text{h}}{4\text{in}} = \frac{74 \times 1}{1 \times 4}\frac{\cancel{\text{in}} \cdot \text{h}}{\cancel{\text{in}}} = 18.5\text{h}$$

So 74 inches is equal to 18.5 hands.

3 Indirect Conversions

Indirect conversions involve a series of direct conversions. The key to solving indirect conversions is to find a chain of units that you can string together to come up with the correct final unit. It isn't quite as hard as it sounds.

Let's say that you are given the conversions listed in the table in section 8, and you want to know how many centimeters there are in 6 feet. As you can see, the table does not contain a conversion directly from feet to centimeters. However, we could convert the feet to inches, and then convert the inches to centimeters. So the path we will follow is ft→in→cm. To get from feet to inches, we use the fact that 1ft=12in, and our two unit multipliers are $\frac{1\text{ft}}{12\text{in}}$ and $\frac{12\text{in}}{1\text{ft}}$. Once again, choose the multiplier that cancels the feet:

$$\frac{6\text{ft}}{1} \times \frac{12\text{in}}{1\text{ft}} = \frac{6 \times 12}{1} \frac{\cancel{\text{ft}} \cdot \text{in}}{\cancel{\text{ft}}} = 72\text{in}$$

To get from inches to centimeters, we use the fact that 1in=2.54cm, and choose from the two unit multipliers $\frac{1\text{in}}{2.54\text{cm}}$ and $\frac{2.54\text{cm}}{1\text{in}}$:

$$\frac{72\text{in}}{1} \times \frac{2.54\text{cm}}{1\text{in}} = \frac{72 \times 2.54}{1} \frac{\cancel{\text{in}} \cdot \text{cm}}{\cancel{\text{in}}} = 182.88\text{cm}$$

Alternatively, we could combine the steps and do it all at once:

$$\frac{6\cancel{\text{ft}}}{1} \times \frac{12\cancel{\text{in}}}{1\cancel{\text{ft}}} \times \frac{2.54\text{cm}}{1\cancel{\text{in}}} = (6 \times 12 \times 2.54)\text{cm} = 182.88\text{cm}$$

4 Squared and Cubed Units

When dealing with units which are squared or cubed (e.g., in^2 or cm^3), you must *be careful to use your unit multipliers the correct number of times.*

For example, if I need to convert 100cm^3 to in^3, and am given that 1in=2.54cm, then I can do one of two things:

1. Cube both sides of the equation 1in=2.54cm, which gives me $1\text{in}^3 = 16.39\text{cm}^3$. Now I can use my new unit multiplier one time:

$$\frac{100\cancel{\text{cm}^3}}{1} \times \frac{1\text{in}^3}{16.39\cancel{\text{cm}^3}} = \frac{100}{16.39}\text{in}^3 = 6.10\text{in}^3$$

2. Use the unit multiplier $\frac{1\text{in}}{2.54\text{cm}}$ three times:

$$\frac{100\text{cm}^3}{1} \times \frac{1\text{in}}{2.54\text{cm}} \times \frac{1\text{in}}{2.54\text{cm}} \times \frac{1\text{in}}{2.54\text{cm}} =$$

$$\frac{100\cancel{\text{cm}^3}}{1} \times \frac{1^3}{2.54^3} \frac{\text{in}^3}{\cancel{\text{cm}^3}} = \frac{100}{16.39}\text{in}^3 = 6.10\text{in}^3$$

Likewise, for squared units, you would either square both sides of your conversion equation, or use the unit multiplier twice.

The above explanation makes sense if you think about it this way: assume that you have a packing box, and you need to find the volume in cubic inches, but the dimensions are printed on the box in centimeters. The volume is length × width × height, so you would have to convert all three dimensions from centimeters to inches before multiplying. That is, you would have to use the unit multiplier that relates inches and centimeters three times.

5 Compound Units

A *compound unit* is one made up of other units. For example, feet per second (ft/s) and pounds per cubic foot (lb/ft^3) are compound units. The process of converting from one compound unit to another simply requires you to use more unit multipliers.

To convert 88 ft/s to mi/hr (miles per hour), I need to convert feet to miles *and* convert seconds to hours. I will go directly from feet to miles using the equation 1mi=5280ft and the corresponding unit multiplier $\frac{1\text{mi}}{5280\text{ft}}$. Then I will go from s→min→hr using the facts that 1hr=60min and 1min=60s. Here it is all in one long step—pay close attention to what cancels:

$$\frac{88\cancel{\text{ft}}}{1\cancel{\text{s}}} \times \frac{1\text{mi}}{5280\cancel{\text{ft}}} \times \frac{60\cancel{\text{s}}}{1\cancel{\text{min}}} \times \frac{60\cancel{\text{min}}}{1\text{hr}} = \frac{88 \times 60 \times 60}{5280}\frac{\text{mi}}{\text{hr}} = 60\frac{\text{mi}}{\text{hr}}$$

Notice how units don't have to be adjacent to cancel. Everything in the numerators gets multiplied together and everything in the denomenators gets multiplied together—the order doesn't matter.

6 Example Problems

Here are some more examples. Remember, practice makes perfect—try to work these yourself before looking at the solutions. I have provided all the conversions you need in Section 8.

1. Convert 12 inches to centimeters.
   Solution
2. Convert 3 yards to inches.
   Solution
3. How many miles is a 10km race?
   Solution
4. How many cubic inches are in a gallon?
   Solution
5. Convert 50 ft/s to m/s.
   Solution
6. How many mL are in a pint (pt)?
   Solution

7 Solutions to Example Problems

1. Look in the table and find the conversion that relates inches and centimeters: 1in=2.54cm. The two possible unit multipliers are

$$\frac{1\text{in}}{2.54\text{cm}} \quad \text{and} \quad \frac{2.54\text{cm}}{1\text{in}}$$

We are trying to get rid of the inches, so we need to use the second multiplier:

$$\frac{12\cancel{\text{in}}}{1} \times \frac{2.54\text{cm}}{1\cancel{\text{in}}} = 12 \times 2.54\text{cm} = 30.48\text{cm}$$

2. Looking in the table, you'll see that there is no equation directly relating yards and inches. However, there is one relating yards and feet, and one relating feet and inches. So our path will be yd→ft→in.

$$\frac{3\cancel{\text{yd}}}{1} \times \frac{3\cancel{\text{ft}}}{1\cancel{\text{yd}}} \times \frac{12\text{in}}{1\cancel{\text{ft}}} = (3 \times 3 \times 12)\text{in} = 108\text{in}$$

3. Because there is no equation in our table which directly relates kilometers and miles, we need to find a chain of conversions which will get us there. We need to follow the path km→m→cm→in→ft→mi.

$$\frac{10\cancel{\text{km}}}{1} \times \frac{1000\cancel{\text{m}}}{1\cancel{\text{km}}} \times \frac{100\cancel{\text{cm}}}{1\cancel{\text{m}}} \times \frac{1\cancel{\text{in}}}{2.54\cancel{\text{cm}}} \times \frac{1\cancel{\text{ft}}}{12\cancel{\text{in}}} \times \frac{1\text{mi}}{5280\cancel{\text{ft}}} = \frac{10 \times 1000 \times 100}{2.54 \times 12 \times 5280}\text{mi} \approx 6.21\text{mi}$$

4. Be very careful here because the gallon is a unit of volume. Again, we look in the table and find that we can convert from gallons to liters. From liters we can convert to milliliters; from milliliters to cubic centimeters; and from cubic centimeters to cubic inches. Let's shorten the conversion process by first cubing the relation between inches and centimeters:

$$(1\text{in})^3 = (2.54\text{cm})^3 \longrightarrow 1\text{in}^3 = 16.39\text{cm}^3$$

Now we can proceed with the conversion—watch the cancellations:

$$\frac{1\cancel{\text{gal}}}{1} \times \frac{3.785\cancel{\text{L}}}{1\cancel{\text{gal}}} \times \frac{1000\cancel{\text{mL}}}{1\cancel{\text{L}}} \times \frac{1\cancel{\text{cm}^3}}{1\cancel{\text{mL}}} \times \frac{1\text{in}^3}{16.39\cancel{\text{cm}^3}} = \frac{3.785 \times 1000}{16.39}\text{in}^3 \approx 231\text{in}^3$$

5. Don't let the fact that these are compound units throw you off. All we are really converting is ft to m, and looking at the table we see that we need to go from feet to inches to centimeters to meters:

$$\frac{50\cancel{\text{ft}}}{1\text{s}} \times \frac{12\cancel{\text{in}}}{1\cancel{\text{ft}}} \times \frac{2.54\cancel{\text{cm}}}{1\cancel{\text{in}}} \times \frac{1\text{m}}{100\cancel{\text{cm}}} = \frac{50 \times 12 \times 2.54}{100} \frac{\text{m}}{\text{s}} = 15.24 \frac{\text{m}}{\text{s}}$$

6. Looking at the conversion table, we see that we need to convert from pints to quarts to gallons to liters to milliliters.

$$\frac{1\cancel{\text{pt}}}{1} \times \frac{1\cancel{\text{qt}}}{2\cancel{\text{pt}}} \times \frac{1\cancel{\text{gal}}}{4\cancel{\text{qt}}} \times \frac{3.785\cancel{\text{L}}}{1\cancel{\text{gal}}} \times \frac{1000\text{mL}}{1\cancel{\text{L}}} = \frac{3.785 \times 1000}{2 \times 4} \text{mL} \approx 473.1\text{mL}$$

8 Table of Conversions

The following table is included for the purpose of illustration—it is far from being a comprehensive table of conversions, but contains all of the conversions necessary to work the problems in this document. Your science book should contain a more comprehensive table.

Length			
1 ft=12 in 1 cm=10 mm	1 in=2.54 cm 1 km=1000 m	1 m=100 cm 1 yd=3 ft	1 mi=5280 ft
Volume			
1 gal=4 qt 1 mL=1 cm^3	1 gal=3.785 L	1 qt=2 pt	1 L=1000 mL
Miscellaneous			
1 kg=2.205 lb[†]	1 N=.225 lb	$180° = \pi$ rad	1 hp=745.7 W

[†] If you are taking physics you should know that this is technically not correct! The kilogram (kg) is a unit of mass, but the pound (lb) is a unit of force. 1 kg only equals 2.205 lb if the acceleration due to gravity is 9.8 m/s^2.